Appendix

Basic Math for Economics

Functions of One Variable

- Variables: The basic elements of algebra, usually called X, Y, and so on, that may be given any numerical value in an equation
- Function:

$$Y = f(X)$$

Independent and Dependent Variables

- **Independent Variable**: a variable that is unaffected by the action of another variable and may be assigned any value
- **Dependent Variable**: a variable whose value is determined by another variable or set of variables

Two Possible Forms of Functional Relationships

- Y is a ***linear function*** of X

$$Y = a + bX$$

  - Table A.1 shows some values of the linear function, Y = 3 + 2X
- Y is a ***nonlinear function*** of X
  - This includes X raised to the powers other than 1
  - Table A.1 shows some values of a quadratic function $Y = -X^2 + 15X$

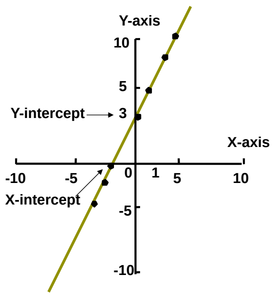
Table A.1: Values of X and Y for Linear and Quadratic Functions

Linear Function		Quadratic Function	
x	$Y = f(X) = 3 + 2X$	x	$Y = f(X) = -X^2 + 15X$
-3	-3	-3	-54
-2	-1	-2	-34
-1	1	-1	-16
0	3	0	0
1	5	1	14
2	7	2	26
3	9	3	36
4	11	4	44
5	13	5	50
6	15	6	54

Graphing Functions of One Variable

- Graphs are used to show the relationship between two variables
- Usually the dependent variable (Y) is shown on the vertical axis and the independent variable (X) is shown on the horizontal axis
  - However, on supply and demand curves, this approach is reversed

Figure A.1: Graph of the Linear Function Y = 3 + 2X



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Intercept

- The general form of a linear equation is
  Y = a + bX
- The Y-**intercept** is the value of Y when X equals 0
  - Using the general form, when X = 0, Y = a, so this is the intercept of the equation

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Slopes

- The **slope** of any straight line is the ratio of the change in Y to the change in X.
- The slope can be defined mathematically as

$$\text{Slope} = \frac{\text{Change in Y}}{\text{Change in X}} = \frac{\Delta Y}{\Delta X}$$

where Δ means “change in”

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Slopes

- For the equation Y = 3 + 2X the slope equals 2 as can be seen in Figure A.1 by the dashed lines representing the changes in X and Y
- As X increases from 0 to 1, Y increases from 3 to 5

$$\text{Slope} = \frac{\Delta Y}{\Delta X} = \frac{5-3}{1-0} = 2$$

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Figure A.1: Graph of the Linear Function Y = 3 + 2X

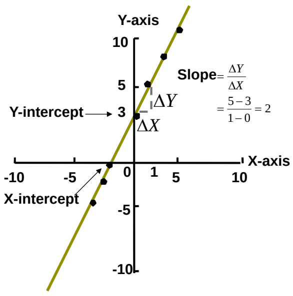

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FIGURE A.2: Changes in the Slope of a Linear Function

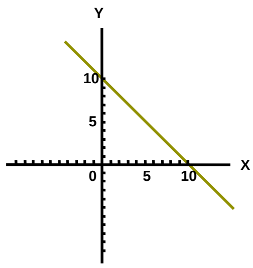

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FIGURE A.2: Changes in the Slope of a Linear Function

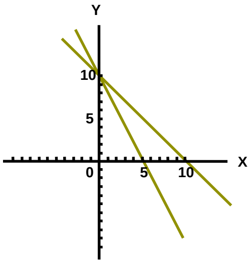

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FIGURE A.3: Changes in the Y-Intercept of a Linear Function

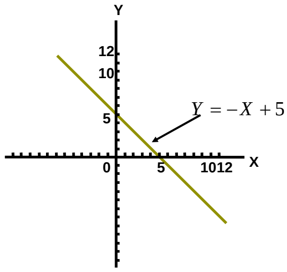

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FIGURE A.3: Changes in the Y-Intercept of a Linear Function

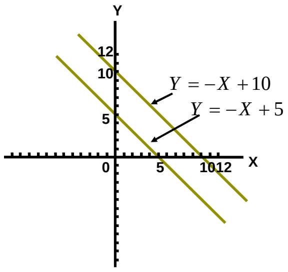

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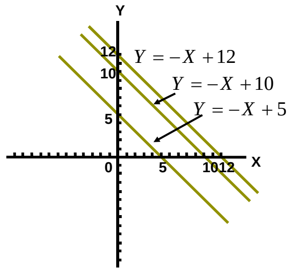
FIGURE A.3: Changes in the Y-Intercept of a Linear Function



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Nonlinear Functions

- Figure A.4 shows the graph of the nonlinear function $Y = -X^2 + 15X$
- The slope of the line is not constant but, in this case, diminishes as X increases
- This results in a concave graph which could reflect the principle of diminishing returns

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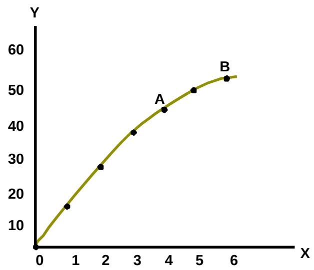
FIGURE A.4: Graph of the Function $Y = X^2 + 15X$



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The Slope of a Nonlinear Function

- The graph of a nonlinear function is not a straight line
- Therefore it does not have the same slope at every point
- The slope of a nonlinear function at a particular point is defined as the slope of the straight line that is tangent to the function at that point.

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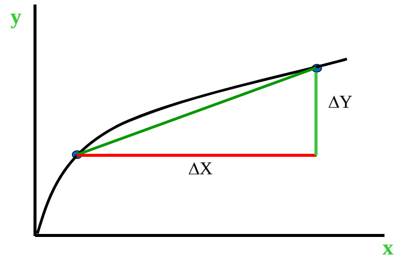

What we have been doing so far is to measure the difference quotient along a chord between two points.

Notice: the change in y relative to the change in x is the slope of the chord (green) line

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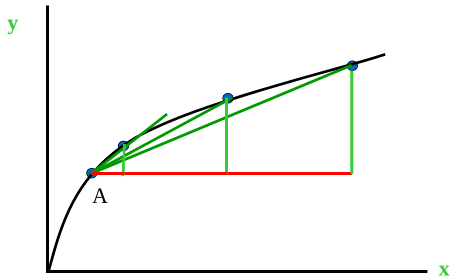

As we reduce the values of x closer to that at point A (ΔX gets closer to zero) the chord becomes more like the tangent. Eventually it will be equal.

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$$

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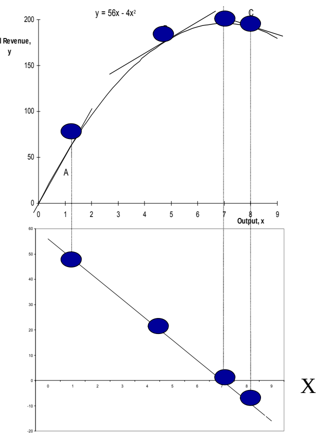
y = f(x)

y = f'(x)



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Functions of Two or More Variables

- The dependent variable can be a function of more than one independent variable
- The general equation for the case where the dependent variable Y is a function of two independent variables X and Z is

$$Y = f(X, Z)$$

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A Simple Example

- Suppose the relationship between the dependent variable (Y) and the two independent variables (X and Z) is given by

$$Y = X \cdot Z$$

- Some values for this function are shown in Table A.2

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TABLE A.2: Values of X, Z, and Y that satisfy the Relationship Y = X·Z

X	Z	Y
1	1	1
1	2	2
1	3	3
1	4	4
2	1	2
2	2	4
2	3	6
2	4	8
3	1	3
3	2	6
3	3	9
3	4	12
4	1	4
4	2	8
4	3	12
4	4	16

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Graphing Functions of Two Variables

- Contour lines are frequently used to graph functions with two independent variables
- Contour lines are lines in two dimensions that show the sets of values of the independent variables that yield the same value for the dependent variable
- Contour lines for the equation Y = X·Z are shown in Figure A.5

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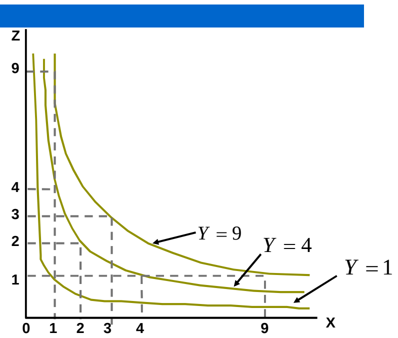
FIGURE A.5: Contour Lines for Y = X·Z



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Derivative of a Function

$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$ is called the derivative of f at a.

We write: $f'(x) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

"The derivative of f with respect to x is …"

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$f'(x)$	"f prime of x" or	"the derivative of f with respect to x"
y'	"y prime"	
$\frac{dy}{dx}$	"dee why dee ecks" or	"the derivative of y with respect to x"
$\frac{df}{dx}$	"dee eff dee ecks" or	"the derivative of f with respect to x"
$\frac{d}{dx} f(x)$	"dee dee ecks uv eff uv ecks" or (d dx of f of x)	"the derivative of f of x"

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Note:

dx does not mean d times x !

dy does not mean d times y !

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Note:

$$\frac{dy}{dx} \text{ does }\underline{\text{not}}\text{ mean } dy \div dx$$

(except when it is convenient to think of it as division.)

$$\frac{df}{dx} \text{ does }\underline{\text{not}}\text{ mean } df \div dx$$

(except when it is convenient to think of it as division.)

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Note:

$$\frac{d}{dx} f(x) \text{ does }\underline{\text{not}}\text{ mean } \frac{d}{dx} \text{ times } f(x)$$

(except when it is convenient to treat it that way.)

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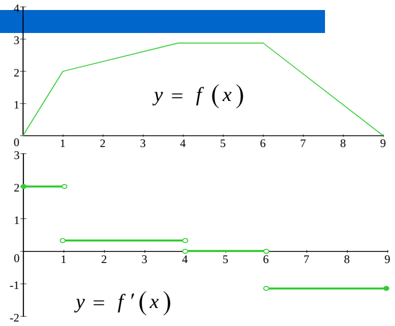
The derivative is the slope of the original function.



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A function is <u>differentiable</u> if it has a derivative everywhere in its domain. It must be <u>continuous</u> and <u>smooth</u>. Functions on closed intervals must have one-sided derivatives defined at the end points.

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Rules of Differentiation

$$\frac{d}{dx}(c) = 0$$

example: $y = 3$

$y' = 0$

The derivative of a constant is zero.

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$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

power rule

examples:

$f(x) = x^4$ $\qquad$ $y = x^8$

$f'(x) = 4x^3$ $\qquad$ $y' = 8x^7$

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constant multiple rule:

$$\frac{d}{dx}(cu) = c\frac{du}{dx}$$

examples:

$$\frac{d}{dx}cx^n = cnx^{n-1}$$

$$\frac{d}{dx}7x^5 = 7\cdot 5x^4 = 35x^4$$

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sum and difference rules:

$$\frac{d}{dx}(u+v) = \frac{du}{dx}+\frac{dv}{dx}$$

$$\frac{d}{dx}(u-v) = \frac{du}{dx}-\frac{dv}{dx}$$

$$y = x^4 + 12x$$

$$y' = 4x^3 + 12$$

$$y = x^4 - 2x^2 + 2$$

$$\frac{dy}{dx} = 4x^3 - 4x$$

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product rule:

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{d}{dx}\left[\left(x^2+3\right)\left(2x^3+5x\right)\right] = \left(x^2+3\right)\left(6x^2+5\right)+\left(2x^3+5x\right)\left(2x\right)$$

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quotient rule:

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$$

$$\frac{d}{dx}\frac{2x^3+5x}{x^2+3} = \frac{\left(x^2+3\right)\left(6x^2+5\right)-\left(2x^3+5x\right)\left(2x\right)}{\left(x^2+3\right)^2}$$

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