Chapter 6
Production

■ Questions for Review

1. **What is a production function? How does a long-run production function differ from a short-run production function?**

   A production function represents how inputs are transformed into outputs by a firm. In particular, a production function describes the *maximum* output that a firm can produce for each specified combination of inputs. In the short run, one or more factors of production cannot be changed, so a short-run production function tells us the maximum output that can be produced with different amounts of the variable inputs, holding fixed inputs constant. In the long-run production function, all inputs are variable.

2. **Why is the marginal product of labor likely to increase initially in the short run as more of the variable input is hired?**

   The marginal product of labor is likely to increase initially because when there are more workers, each is able to specialize in an aspect of the production process in which he or she is particularly skilled. For example, think of the typical fast food restaurant. If there is only one worker, he will need to prepare the burgers, fries, and sodas, as well as take the orders. Only so many customers can be served in an hour. With two or three workers, each is able to specialize, and the marginal product (number of customers served per hour) is likely to increase as we move from one to two to three workers. Eventually, there will be enough workers and there will be no more gains from specialization. At this point, the marginal product will begin to diminish.

3. **Why does production eventually experience diminishing marginal returns to labor in the short run?**

   The marginal product of labor will eventually diminish because there will be at least one fixed factor of production, such as capital. As more and more labor is used along with a fixed amount of capital, there is less and less capital for each worker to use, and the productivity of additional workers necessarily declines. Think for example of an office where there are only three computers. As more and more employees try to share the computers, the marginal product of each additional employee will diminish.

4. **You are an employer seeking to fill a vacant position on an assembly line. Are you more concerned with the average product of labor or the marginal product of labor for the last person hired? If you observe that your average product is just beginning to decline, should you hire any more workers? What does this situation imply about the marginal product of your last worker hired?**

   In filling a vacant position, you should be concerned with the marginal product of the last worker hired, because the marginal product measures the effect on output, or total product, of hiring another worker. This in turn determines the additional revenue generated by hiring another worker, which should then be compared to the cost of hiring the additional worker.

The point at which the average product begins to decline is the point where average product is equal to marginal product. As more workers are used beyond this point, both average product and marginal product decline. However, marginal product is still positive, so total product continues to increase. Thus, it may still be profitable to hire another worker.

5. **What is the difference between a production function and an isoquant?**

A production function describes the maximum output that can be achieved with any given combination of inputs. An isoquant identifies all of the different combinations of inputs that can be used to produce one particular level of output.

6. **Faced with constantly changing conditions, why would a firm ever keep *any* factors fixed? What criteria determine whether a factor is fixed or variable?**

Whether a factor is fixed or variable depends on the time horizon under consideration: all factors are fixed in the very short run while all factors are variable in the long run. As stated in the text, “All fixed inputs in the short run represent outcomes of previous long-run decisions based on estimates of what a firm could profitably produce and sell.” Some factors are fixed in the short run, whether the firm likes it or not, simply because it takes time to adjust the levels of those inputs. For example, a lease on a building may legally bind the firm, some employees may have contracts that must be upheld, or construction of a new facility may take a year or more. Recall that the short run is not defined as a specific number of months or years but as that period of time during which some inputs cannot be changed for reasons such as those given above.

7. **Isoquants can be convex, linear, or L-shaped. What does each of these shapes tell you about the nature of the production function? What does each of these shapes tell you about the *MRTS*?**

Convex isoquants indicate that some units of one input can be substituted for a unit of the other input while maintaining output at the same level. In this case, the *MRTS* is diminishing as we move down along the isoquant. This tells us that it becomes more and more difficult to substitute one input for the other while keeping output unchanged. Linear isoquants imply that the slope, or the *MRTS*, is constant. This means that the same number of units of one input can always be exchanged for a unit of the other input holding output constant. The inputs are perfect substitutes in this case. L-shaped isoquants imply that the inputs are perfect complements, and the firm is producing under a fixed proportions type of technology. In this case the firm cannot give up one input in exchange for the other and still maintain the same level of output. For example, the firm may require exactly 4 units of capital for each unit of labor, in which case one input cannot be substituted for the other.

8. **Can an isoquant ever slope upward? Explain.**

No. An upward sloping isoquant would mean that if you increased both inputs output would stay the same. This would occur only if one of the inputs reduced output; sort of like a bad in consumer theory. As a general rule, if the firm has more of all inputs it can produce more output.

9. **Explain the term “marginal rate of technical substitution.” What does a *MRTS* = 4 mean?**

MRTS is the amount by which the quantity of one input can be reduced when the other input is increased by one unit, while maintaining the same level of output. If the *MRTS* is 4 then one input can be reduced by 4 units as the other is increased by one unit, and output will remain the same.

10. **Explain why the marginal rate of technical substitution is likely to diminish as more and more labor is substituted for capital.**

As more and more labor is substituted for capital, it becomes increasingly difficult for labor to perform the jobs previously done by capital. Therefore, more units of labor will be required to replace each unit of capital, and the *MRTS* will diminish. For example, think of employing more and more farm labor while reducing the number of tractor hours used. At first you would stop using tractors for simpler tasks such as driving around the farm to examine and repair fences or to remove rocks and fallen tree limbs from fields. But eventually, as the number or labor hours increased and the number of tractor hours declined, you would have to plant and harvest your crops primarily by hand. This would take huge numbers of additional workers.

11. Is it possible to have diminishing returns to a single factor of production and constant returns to scale at the same time? Discuss.

Diminishing returns and returns to scale are completely different concepts, so it is quite possible to have both diminishing returns to, say, labor and constant returns to scale. Diminishing returns to a single factor occurs because all other inputs are fixed. Thus, as more and more of the variable factor is used, the additions to output eventually become smaller and smaller because there are no increases in the other factors. The concept of returns to scale, on the other hand, deals with the increase in output when *all* factors are increased by the same proportion. While each factor by itself exhibits diminishing returns, output may more than double, less than double, or exactly double when all the factors are doubled. The distinction again is that with returns to scale, all inputs are increased in the same proportion and no inputs are fixed. The production function in Exercise 10 is an example of a function with diminishing returns to each factor and constant returns to scale.

12. Can a firm have a production function that exhibits increasing returns to scale, constant returns to scale, and decreasing returns to scale as output increases? Discuss.

Many firms have production functions that exhibit first increasing, then constant, and ultimately decreasing returns to scale. At low levels of output, a proportional increase in all inputs may lead to a larger-than-proportional increase in output, because there are many ways to take advantage of greater specialization as the scale of operation increases. As the firm grows, the opportunities for specialization may diminish, and the firm operates at peak efficiency. If the firm wants to double its output, it must duplicate what it is already doing. So it must double all inputs in order to double its output, and thus there are constant returns to scale. At some level of production, the firm will be so large that when inputs are doubled, output will less than double, a situation that can arise from management diseconomies.

13. Suppose that output q is a function of a single input, labor (L). Describe the returns to scale associated with each of the following production functions:

a. **$q = L/2$.** Let q' be output when labor is doubled to $2L$. Then $q' = (2L)/2 = L$. Compare q' to q by dividing q' by q. This gives us $q'/q = L/(L/2) = 2$. Therefore when the amount of labor is doubled, output is also doubled. Hence there are constant returns to scale.

b. **$q = L^2 + L$.** Again, let q' be output when labor is doubled. $q' = (2L)^2 + 2L = 4L^2 + 2L$. Dividing by q yields $q'/q = (4L^2 + 2L)/(L^2 + L) > 2$. To see why this ratio is greater than two, note that it would be exactly two if q' were to equal $2L^2 + 2L$, but q' is larger than that, so the ratio is greater than two, indicating increasing returns to scale.

c. **$q = \log(L)$.** In this case, $q' = \log(2L) = \log(2) + \log(L)$, using the rules for logarithms. Then $q'/q = [\log(2) + \log(L)]/\log(L) = \log(2)/\log(L) + 1$. This expression is greater than, equal to or less than 2 when L is less than, equal to or greater than 2. So this production function exhibits increasing returns to scale when $L < 2$, constant returns to scale when $L = 2$, and decreasing returns to scale when $L > 2$.

■ Exercises

1. **The menu at Joe's coffee shop consists of a variety of coffee drinks, pastries, and sandwiches. The marginal product of an additional worker can be defined as the number of customers who can be served by that worker in a given time period. Joe has been employing one worker, but is considering hiring a second and a third. Explain why the marginal product of the second and third workers might be higher than the first. Why might you expect the marginal product of additional workers to diminish eventually?**

   The marginal product could well increase for the second and third workers because each would be able to specialize in a different task. If there is only one worker, that person has to take orders, prepare the food, serve the food, and do all the cleanup. With two or three workers, however, one could take orders and serve the food while the others do most of the coffee and food preparation and cleanup. Eventually, however, as more workers are employed, the marginal product would diminish because there would be a large number of people behind the counter and in the kitchen trying to serve more and more customers with a limited amount of equipment and a fixed building size.

2. **Suppose a chair manufacturer is producing in the short run (with its existing plant and equipment). The manufacturer has observed the following levels of production corresponding to different numbers of workers:**

Number of Workers	Number of Chairs
1	10
2	18
3	24
4	28
5	30
6	28
7	25

   a. **Calculate the marginal and average product of labor for this production function.**

   The average product of labor, AP_L, is equal to $\frac{q}{L}$. The marginal product of labor, MP_L, is equal to $\frac{\Delta q}{\Delta L}$, the change in output divided by the change in labor input. For this production process we have:

L	q	AP_L	MP_L
0	0	—	—
1	10	10	10
2	18	9	8
3	24	8	6
4	28	7	4
5	30	6	2
6	28	4.7	–2
7	25	3.6	–3

b. Does this production function exhibit diminishing returns to labor? Explain.

Yes, this production function exhibits diminishing returns to labor. The marginal product of labor, the extra output produced by each additional worker, diminishes as workers are added, and this starts to occur with the second unit of labor.

c. Explain intuitively what might cause the marginal product of labor to become negative.

Labor's negative marginal product for $L > 5$ may arise from congestion in the chair manufacturer's factory. Since more laborers are using the same fixed amount of capital, it is possible that they could get in each other's way, decreasing efficiency and the amount of output. Firms also have to control the quality of their output, and the high congestion of labor may produce products that are not of a high enough quality to be offered for sale, which can contribute to a negative marginal product.

3. Fill in the gaps in the table below.

Quantity of Variable Input	Total Output	Marginal Product of Variable Input	Average Product of Variable Input
0	0	—	—
1	225		
2			300
3		300	
4	1140		
5		225	
6			225

Quantity of Variable Input	Total Output	Marginal Product of Variable Input	Average Product of Variable Input
0	0	—	—
1	225	225	225
2	600	375	300
3	900	300	300
4	1140	240	285
5	1365	225	273
6	1350	–15	225

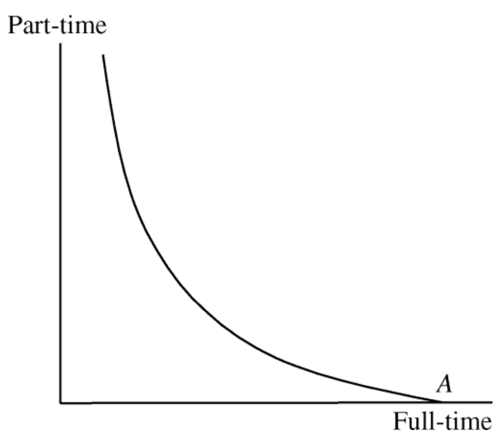
4. A political campaign manager must decide whether to emphasize television advertisements or letters to potential voters in a reelection campaign. Describe the production function for campaign votes. How might information about this function (such as the shape of the isoquants) help the campaign manager to plan strategy?

The output of concern to the campaign manager is the number of votes. The production function has two inputs, television advertising and letters. The use of these inputs requires knowledge of the substitution possibilities between them. If the inputs are perfect substitutes for example, the isoquants are straight lines, and the campaign manager should use only the less expensive input in this case. If the inputs are not perfect substitutes, the isoquants will have a convex shape. The campaign manager should then spend the campaign's budget on the combination of the two inputs that maximize the number of votes.

5. For each of the following examples, draw a representative isoquant. What can you say about the marginal rate of technical substitution in each case?

a. A firm can hire only full-time employees to produce its output, or it can hire some combination of full-time and part-time employees. For each full-time worker let go, the firm must hire an increasing number of temporary employees to maintain the same level of output.

Place part-time workers on the vertical axis and full-time workers on the horizontal. The slope of the isoquant measures the number of part-time workers that can be exchanged for a full-time worker while still maintaining output. At the bottom end of the isoquant, at point *A*, the isoquant hits the full-time axis because it is possible to produce with full-time workers only and no part-timers. As we move up the isoquant and give up full-time workers, we must hire more and more part-time workers to replace each full-time worker. The slope increases (in absolute value) as we move up the isoquant. The isoquant is therefore convex and there is a diminishing marginal rate of technical substitution.



b. A firm finds that it can always trade two units of labor for one unit of capital and still keep output constant.

The marginal rate of technical substitution measures the number of units of capital that can be exchanged for a unit of labor while still maintaining output. If the firm can always trade two units of labor for one unit of capital then the *MRTS* of labor for capital is constant and equal to 1/2, and the isoquant is linear.

c. A firm requires exactly two full-time workers to operate each piece of machinery in the factory.

This firm operates under a fixed proportions technology, and the isoquants are L-shaped. The firm cannot substitute any labor for capital and still maintain output because it must maintain a fixed 2:1 ratio of labor to capital. The *MRTS* is infinite (or undefined) along the vertical part of the isoquant and zero on the horizontal part.

6. A firm has a production process in which the inputs to production are perfectly substitutable in the long run. Can you tell whether the marginal rate of technical substitution is high or low, or is further information necessary? Discuss.

Further information is necessary. The marginal rate of technical substitution, *MRTS*, is the absolute value of the slope of an isoquant. If the inputs are perfect substitutes, the isoquants will be linear. To calculate the slope of the isoquant, and hence the *MRTS*, we need to know the rate at which one input may be substituted for the other. In this case, we do not know whether the *MRTS* is high or low. All we know is that it is a constant number. We need to know the marginal product of each input to determine the *MRTS*.

7. The marginal product of labor in the production of computer chips is 50 chips per hour. The marginal rate of technical substitution of hours of labor for hours of machine capital is 1/4. What is the marginal product of capital?

The marginal rate of technical substitution is defined at the ratio of the two marginal products. Here, we are given the marginal product of labor and the marginal rate of technical substitution. To determine the marginal product of capital, substitute the given values for the marginal product of labor and the marginal rate of technical substitution in the following formula:

$$\frac{MP_L}{MP_K} = MRTS, \text{ or } \frac{50}{MP_K} = \frac{1}{4},$$

and therefore, $MP_K = 200$ computer chips per hour.

8. Do the following functions exhibit increasing, constant, or decreasing returns to scale? What happens to the marginal product of each individual factor as that factor is increased and the other factor held constant?

a. $q = 3L + 2K$

This function exhibits constant returns to scale. For example, if L is 2 and K is 2 then q is 10. If L is 4 and K is 4 then q is 20. When the inputs are doubled, output will double. Each marginal product is constant for this production function. When L increases by 1, q will increase by 3. When K increases by 1, q will increase by 2.

b. $q = (2L + 2K)^{\frac{1}{2}}$

This function exhibits decreasing returns to scale. For example, if L is 2 and K is 2 then q is 2.8. If L is 4 and K is 4 then q is 4. When the inputs are doubled, output increases by less than double. The marginal product of each input is decreasing. This can be determined using calculus by differentiating the production function with respect to one input while holding the other input constant. For example, the marginal product of labor is

$$\frac{\partial q}{\partial L} = \frac{2}{2(2L + 2K)^{\frac{1}{2}}}.$$

Since L is in the denominator, as L gets bigger, the marginal product gets smaller. If you do not know calculus, you can choose several values for L (holding K fixed at some level), find the corresponding q values and see how the marginal product changes. For example, if $L = 4$ and $K = 4$ then $q = 4$. If $L = 5$ and $K = 4$ then $q = 4.24$. If $L = 6$ and $K = 4$ then $q = 4.47$. Marginal product of labor falls from 0.24 to 0.23. Thus, MP_L decreases as L increases, holding K constant at 4 units.

c. $q = 3LK^2$

This function exhibits increasing returns to scale. For example, if L is 2 and K is 2, then q is 24. If L is 4 and K is 4 then q is 192. When the inputs are doubled, output more than doubles. Notice also that if we increase each input by the same factor λ then we get the following:

$$q' = 3(\lambda L)(\lambda K)^2 = \lambda^3 3LK^2 = \lambda^3 q.$$

Since λ is raised to a power greater than 1, we have increasing returns to scale.

The marginal product of labor is constant and the marginal product of capital is increasing. For any given value of K, when L is increased by 1 unit, q will go up by $3K^2$ units, which is a constant number. Using calculus, the marginal product of capital is $MP_K = 6LK$. As K increases, MP_K increases. If you do not know calculus, you can fix the value of L, choose a starting value for K, and find q. Now increase K by 1 unit and find the new q. Do this a few more times and you can calculate marginal product. This was done in part b above, and in part d below.

d. $q = L^{\frac{1}{2}} K^{\frac{1}{2}}$

This function exhibits constant returns to scale. For example, if L is 2 and K is 2 then q is 2. If L is 4 and K is 4 then q is 4. When the inputs are doubled, output will exactly double. Notice also that if we increase each input by the same factor, λ, then we get the following:

$$q' = (\lambda L)^{\frac{1}{2}} (\lambda K)^{\frac{1}{2}} = \lambda L^{\frac{1}{2}} K^{\frac{1}{2}} = \lambda q.$$

Since λ is raised to the power 1, there are constant returns to scale.

The marginal product of labor is decreasing and the marginal product of capital is decreasing. Using calculus, the marginal product of capital is

$$MP_K = \frac{L^{\frac{1}{2}}}{2K^{\frac{1}{2}}}.$$

For any given value of L, as K increases, MP_K will decrease. If you do not know calculus then you can fix the value of L, choose a starting value for K, and find q. Let $L = 4$ for example. If K is 4 then q is 4, if K is 5 then q is 4.47, and if K is 6 then q is 4.90. The marginal product of the 5th unit of K is $4.47 - 4 = 0.47$, and the marginal product of the 6th unit of K is $4.90 - 4.47 = 0.43$. Hence we have diminishing marginal product of capital. You can do the same thing for labor.

e. $q = 4L^{\frac{1}{2}} + 4K$

This function exhibits decreasing returns to scale. For example, if L is 2 and K is 2 then q is 13.66. If L is 4 and K is 4 then q is 24. When the inputs are doubled, output increases by less than double.

The marginal product of labor is decreasing and the marginal product of capital is constant. For any given value of L, when K is increased by 1 unit, q goes up by 4 units, which is a constant number. To see that the marginal product of labor is decreasing, fix $K = 1$ and choose values for L. If $L = 1$ then $q = 8$, if $L = 2$ then $q = 9.66$, and if $L = 3$ then $q = 10.93$. The marginal product of the second unit of labor is $9.66 - 8 = 1.66$, and the marginal product of the third unit of labor is $10.93 - 9.66 = 1.27$. Marginal product of labor is diminishing.

9. The production function for the personal computers of DISK, Inc., is given by $q = 10K^{0.5}L^{0.5}$, where q is the number of computers produced per day, K is hours of machine time, and L is hours of labor input. DISK's competitor, FLOPPY, Inc., is using the production function $q = 10K^{0.6}L^{0.4}$.

a. If both companies use the same amounts of capital and labor, which will generate more output?

Let q_1 be the output of DISK, Inc., q_2, be the output of FLOPPY, Inc., and X be the same equal amounts of capital and labor for the two firms. Then according to their production functions,

$$q_1 = 10X^{0.5}X^{0.5} = 10X^{(0.5 + 0.5)} = 10X$$

and

$$q_2 = 10X^{0.6}X^{0.4} = 10X^{(0.6 + 0.4)} = 10X.$$

Because $q_1 = q_2$, both firms generate the same output with the same inputs. Note that if the two firms both used the same amount of capital and the same amount of labor, but the amount of capital was not equal to the amount of labor, then the two firms would not produce the same levels of output. In fact, if $K > L$ then $q_2 > q_1$, and if $L > K$ then $q_1 > q_2$.

b. Assume that capital is limited to 9 machine hours, but labor is unlimited in supply. In which company is the marginal product of labor greater? Explain.

With capital limited to 9 machine hours, the production functions become $q_1 = 30L^{0.5}$ and $q_2 = 37.37L^{0.4}$. To determine the production function with the highest marginal productivity of labor, consider the following table:

L	q Firm 1	MP_L Firm 1	q Firm 2	MP_L Firm 2
0	0.0	—	0.00	—
1	30.00	30.00	37.37	37.37
2	42.43	12.43	49.31	11.94
3	51.96	9.53	57.99	8.68
4	60.00	8.04	65.06	7.07

For each unit of labor above 1, the marginal productivity of labor is greater for the first firm, DISK, Inc.

10. In Example 6.4, wheat is produced according to the production function $q = 100(K^{0.8}L^{0.2})$.

a. Beginning with a capital input of 4 and a labor input of 49, show that the marginal product of labor and the marginal product of capital are both decreasing.

For fixed labor and variable capital:

$K = 4 \Rightarrow q = (100)(4^{0.8})(49^{0.2}) = 660.22$

$K = 5 \Rightarrow q = (100)(5^{0.8})(49^{0.2}) = 789.25 \Rightarrow MP_K = 129.03$

$K = 6 \Rightarrow q = (100)(6^{0.8})(49^{0.2}) = 913.19 \Rightarrow MP_K = 123.94$

$K = 7 \Rightarrow q = (100)(7^{0.8})(49^{0.2}) = 1033.04 \Rightarrow MP_K = 119.85.$

For fixed capital and variable labor:

$L = 49 \Rightarrow q = (100)(4^{0.8})(49^{0.2}) = 660.22$

$L = 50 \Rightarrow q = (100)(4^{0.8})(50^{0.2}) = 662.89 \Rightarrow MP_L = 2.67$

$L = 51 \Rightarrow q = (100)(4^{0.8})(51^{0.2}) = 665.52 \Rightarrow MP_L = 2.63$

$L = 52 \Rightarrow q = (100)(4^{0.8})(52^{0.2}) = 668.11 \Rightarrow MP_L = 2.59.$

The marginal products of both capital and labor decrease as the variable input increases.

b. Does this production function exhibit increasing, decreasing, or constant returns to scale?

Constant (increasing, decreasing) returns to scale implies that proportionate increases in inputs lead to the same (more than, less than) proportionate increases in output. If we were to increase labor and capital by the same proportionate amount (λ) in this production function, output would change by the same proportionate amount:

$$q' = 100(\lambda K)^{0.8}(\lambda L)^{0.2}, \text{ or}$$
$$q' = 100K^{0.8}L^{0.2}\lambda^{(0.8+0.2)} = q\lambda$$

Therefore this production function exhibits constant returns to scale. You can also determine this if you plug in values for K and L and compute q, and then double the K and L values to see what happens to q. For example, let $K = 4$ and $L = 10$. Then $q = 480.45$. Now double both inputs to $K = 8$ and $L = 20$. The new value for q is 960.90, which is exactly twice as much output. Thus there are constant returns to scale.

11. Suppose life expectancy in years (*L*) is a function of two inputs, health expenditures (*H*) and nutrition expenditures (*N*) in hundreds of dollars per year. The production function is $L = cH^{0.8}N^{0.2}$.

a. Beginning with a health input of \$400 per year ($H = 4$) and a nutrition input of \$4900 per year ($N = 49$), show that the marginal product of health expenditures and the marginal product of nutrition expenditures are both decreasing.

When $H = 4$ and $N = 49$, $L = c(4^{0.8})(49^{0.2}) = 6.602c$. Holding N constant at 49, when $H = 5$, $L = 7.893c$, and when $H = 6$, $L = 9.132c$. The marginal product of H drops from 1.291c (7.893c − 6.602c) to 1.239c (9.132c − 7.893c). Therefore the marginal product of health expenditures is decreasing.

Now hold H constant at 4 and increase N to 50. $L = c(4^{0.8})(50^{0.2}) = 6.629c$. Increasing N to 51, $L = 6.655c$. The marginal product of N drops from 0.027c (6.629c − 6.602c) to 0.026c (6.655c − 6.629c), so the marginal product of nutrition expenditures is decreasing.

b. Does this production function exhibit increasing, decreasing, or constant returns to scale?

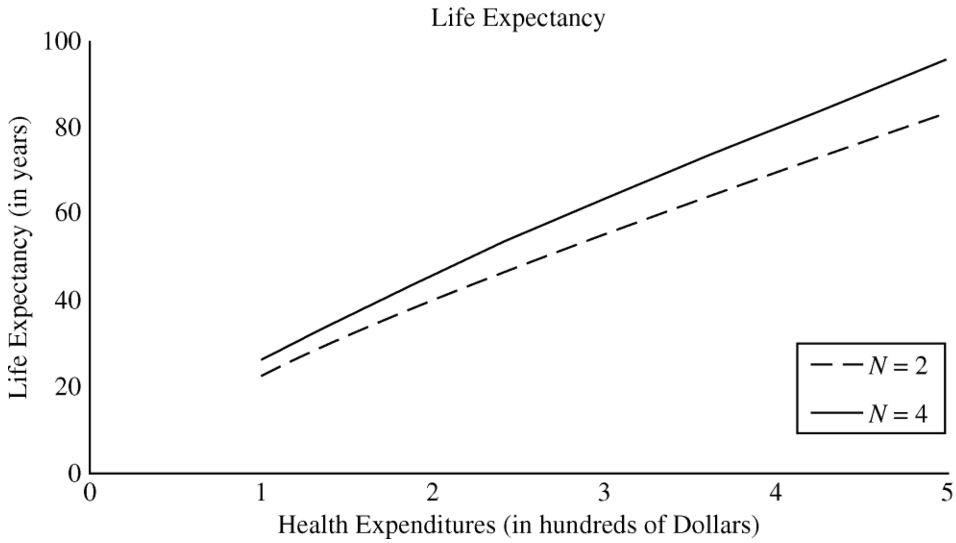
There are constant returns to scale. If both inputs are doubled, the new output is $L' = c(2H)^{0.8}(2N)^{0.2} = cH^{0.8}N^{0.2}(2^{0.8})(2^{0.2}) = 2cH^{0.8}N^{0.2} = 2L$. So when both inputs are doubled, life expectancy also doubles. Hence there are constant returns to scale.

c. Suppose that in a country suffering from famine, *N* is fixed at 2 and that *c* = 20. Plot the production function for life expectancy as a function of health expenditures, with *L* on the vertical axis and *H* on the horizontal axis.

The production function becomes $L = 20(H^{0.8})(2^{0.2}) = 22.974H^{0.8}$. To plot this, find the life expectancies for various levels of H, plot those points and draw a smooth curve through them. Here are some points: $H = 1$ and $L = 22.97$; $H = 2$ and $L = 40.00$; $H = 3$ and $L = 55.38$; $H = 4$ and $L = 69.64$; $H = 5$ and $L = 83.26$. This production function is plotted below as a dashed line.

d. Now suppose another nation provides food aid to the country suffering from famine so that *N* increases to 4. Plot the new production function.

The production function becomes $L = 20(H^{0.8})(4^{0.2}) = 26.390H^{0.8}$. Points to plot are $H = 1$ and $L = 26.39$; $H = 2$ and $L = 45.95$; $H = 3$ and $L = 63.55$; $H = 4$ and $L = 80.00$; $H = 5$ and $L = 95.63$. This production function is plotted below as a solid line.



e. Now suppose that *N* = 4 and *H* = 2. You run a charity that can provide either food aid or health aid to this country. Which would provide a greater benefit: increasing *H* by 1 or *N* by 1?

If $N = 4$ and $H = 2$, life expectancy is 45.95 years as calculated in part d. If N remains at 4 and H increases by 1 (so $H = 3$), life expectancy increases to 63.55 years as shown in part d. On the other hand, if H remains at 2 and N increases by 1 (so $N = 5$), life expectancy rises to $20(2^{0.8})(5^{0.2}) = 48.05$. It is clearly much more beneficial to increase H by 1, because life expectancy increases to 63.55 years. An increase in N by 1 raises life expectancy to only 48.05 years.