Chapter 7
The Cost of Production

■ Questions for Review

1. A firm pays its accountant an annual retainer of $10,000. Is this an economic cost?

This is an explicit cost of purchasing the services of the accountant, and it is both an economic and an accounting cost. When the firm pays an annual retainer of $10,000, there is a monetary transaction. The accountant trades his or her time in return for money. An annual retainer is an explicit cost and therefore an economic cost.

2. The owner of a small retail store does her own accounting work. How would you measure the opportunity cost of her work?

The economic, or opportunity, cost of doing accounting work is measured by computing the monetary amount that the owner's time would be worth in its *next best use*. For example, if she could do accounting work for some other company instead of her own, her opportunity cost is the amount she could have earned in that alternative employment. Or if she is a great stand-up comic, her opportunity cost is what she could have earned in that occupation instead of doing her own accounting work.

3. Please explain whether the following statements are true or false.

a. If the owner of a business pays himself no salary, then the accounting cost is zero, but the economic cost is positive.

True. Since there is no monetary transaction, there is no accounting, or explicit, cost. However, since the owner of the business could be employed elsewhere, there is an economic cost. The economic cost is positive, reflecting the opportunity cost of the owner's time. The economic cost is the value of the owner's time in his next best alternative, or the amount that the owner would earn if he took the next best job.

b. A firm that has positive accounting profit does not necessarily have positive economic profit.

True. Accounting profit considers only the explicit, monetary costs. Since there may be some opportunity costs that were not fully realized as explicit monetary costs, it is possible that when the opportunity costs are added in, economic profit will become negative. This indicates that the firm's resources are not being put to their best use.

c. If a firm hires a currently unemployed worker, the opportunity cost of utilizing the worker's services is zero.

False. From the firm's point of view, the wage paid to the worker is an explicit cost whether she was previously unemployed or not. The firm's opportunity cost is equal to the wage, because if it did not hire this worker, it would have had to hire someone else at the same wage. The opportunity cost from the worker's point of view is the value of her time, which is unlikely to be zero. By taking this job, she cannot work at another job or take care of a child or elderly person at home. If her best alternative is working at another job, she gives up the wage she would have earned.

If her best alternative is unpaid, such as taking care of a loved one, she will now have to pay someone else to do that job, and the amount she has to pay is her opportunity cost.

4. **Suppose that labor is the only variable input to the production process. If the marginal cost of production is diminishing as more units of output are produced, what can you say about the marginal product of labor?**

   The marginal product of labor must be increasing. The marginal cost of production measures the extra cost of producing one more unit of output. If this cost is diminishing, then it must be taking fewer units of labor to produce the extra unit of output. If fewer units of labor are required to produce a unit of output, then the marginal product (extra output produced by an extra unit of labor) must be increasing. Note also, that $MC = w/MP_L$, so that if MC is diminishing then MP_L must be increasing for any given w.

5. **Suppose a chair manufacturer finds that the marginal rate of technical substitution of capital for labor in her production process is substantially greater than the ratio of the rental rate on machinery to the wage rate for assembly-line labor. How should she alter her use of capital and labor to minimize the cost of production?**

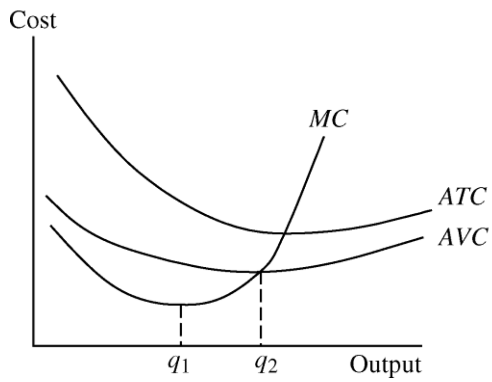
   The question states that the *MRTS* of capital for labor is greater than r/w. Note that this is different from the *MRTS* of labor for capital, which is what is used in Chapters 6 and 7. The *MRTS* of labor for capital equals MP_K/MP_L. So it follows that $MP_K/MP_L > r/w$ or, written another way, $MP_K/r > MP_L/w$. These two ratios should be equal to minimize cost. Since the manufacturer gets more marginal output per dollar from capital than from labor, she should use more capital and less labor to minimize the cost of production.

6. **Why are isocost lines straight lines?**

   The isocost line represents all possible combinations of two inputs that may be purchased for a given total cost. The slope of the isocost line is the negative of the ratio of the input prices. If the input prices are fixed, their ratio is constant and the isocost line is therefore straight. Only if the ratio of the input prices changes as the quantities of the inputs change is the isocost line not straight.

7. **Assume that the marginal cost of production is increasing. Can you determine whether the average variable cost is increasing or decreasing? Explain.**

   No. When marginal cost is increasing, average variable cost can be either increasing or decreasing as shown in the diagram below. Marginal cost begins increasing at output level q_1, but *AVC* is decreasing. This happens because *MC* is below *AVC* and is therefore pulling *AVC* down. *AVC* is decreasing for all output levels between q_1 and q_2. At q_2, *MC* cuts through the minimum point of *AVC*, and *AVC* begins to rise because *MC* is above it. Thus for output levels greater than q_2, *AVC* is increasing.



8. Assume that the marginal cost of production is greater than the average variable cost. Can you determine whether the average variable cost is increasing or decreasing? Explain.

Yes, the average variable cost is increasing. If marginal cost is above average variable cost, each additional unit costs more to produce than the average of the previous units, so the average variable cost is pulled upward. This is shown in the diagram above for output levels greater than q_2.

9. If the firm's average cost curves are U-shaped, why does its average variable cost curve achieve its minimum at a lower level of output than the average total cost curve?

Average total cost is equal to average fixed cost plus average variable cost: $ATC = AVC + AFC$. When graphed, the difference between the U-shaped average total cost and U-shaped average variable cost curves is the average fixed cost, and *AFC* is downward sloping at all output levels. When *AVC* is falling, *ATC* will also fall because both *AVC* and *AFC* are declining as output increases. When *AVC* reaches its minimum (the bottom of its U), *ATC* will continue to fall because *AFC* is falling. Even as *AVC* gradually begins to rise, *ATC* will still fall because of *AFC*'s decline. Eventually, however, as *AVC* rises more rapidly, the increases in *AVC* will outstrip the declines in *AFC*, and *ATC* will reach its minimum and then begin to rise.

10. If a firm enjoys economies of scale up to a certain output level, and cost then increases proportionately with output, what can you say about the shape of the long-run average cost curve?

When the firm experiences economies of scale, its long-run average cost curve is downward sloping. When costs increase proportionately with output, the firm's long-run average cost curve is horizontal. So this firm's long-run average cost curve has a rounded L-shape; first it falls and then it becomes horizontal as output increases.

11. How does a change in the price of one input change the firm's long-run expansion path?

The expansion path describes the cost-minimizing combination of inputs that the firm chooses for every output level. This combination depends on the ratio of input prices, so if the price of one input changes, the price ratio also changes. For example, if the price of an input increases, the intercept of the isocost line on that input's axis moves closer to the origin, and the slope of the isocost line (the price ratio) changes. As the price ratio changes, the firm substitutes away from the now more expensive input toward the cheaper input. Thus the expansion path bends toward the axis of the now cheaper input.

12. Distinguish between economies of scale and economies of scope. Why can one be present without the other?

Economies of scale refer to the production of *one* good and occur when total cost increases by a smaller proportion than output. Economies of scope refer to the production of *two or more goods* and occur when joint production is less costly than the sum of the costs of producing each good separately. There is no direct relationship between economies of scale and economies of scope, so production can exhibit one without the other. For example, there are economies of scale producing computers and economies of scale producing carpeting, but if one company produced both, there would likely be no synergies associated with joint production and hence no economies of scope.

13. Is the firm's expansion path always a straight line?

No. If the firm always uses capital and labor in the same proportion, the long run expansion path is a straight line. But if the optimal capital-labor ratio changes as output is increased, the expansion path is not a straight line.

14. What is the difference between economies of scale and returns to scale?

Economies of scale depend on the relationship between cost and output—i.e., how does cost change when output is doubled? Returns to scale depend on what happens to output when all inputs are doubled. The difference is that economies of scale reflect input proportions that change optimally as output is increased, while returns to scale are based on fixed input proportions (such as two units of labor for every unit of capital) as output increases.

■ Exercises

1. Joe quits his computer programming job, where he was earning a salary of $50,000 per year, to start his own computer software business in a building that he owns and was previously renting out for $24,000 per year. In his first year of business he has the following expenses: salary paid to himself, $40,000; rent, $0; other expenses, $25,000. Find the accounting cost and the economic cost associated with Joe's computer software business.

The accounting cost includes only the explicit expenses, which are Joe's salary and his other expenses: $40,000 + 25,000 = $65,000. Economic cost includes these explicit expenses plus opportunity costs. Therefore, economic cost includes the $24,000 Joe gave up by not renting the building and an extra $10,000 because he paid himself a salary $10,000 below market ($50,000 – 40,000). Economic cost is then $40,000 + 25,000 + 24,000 + 10,000 = $99,000.

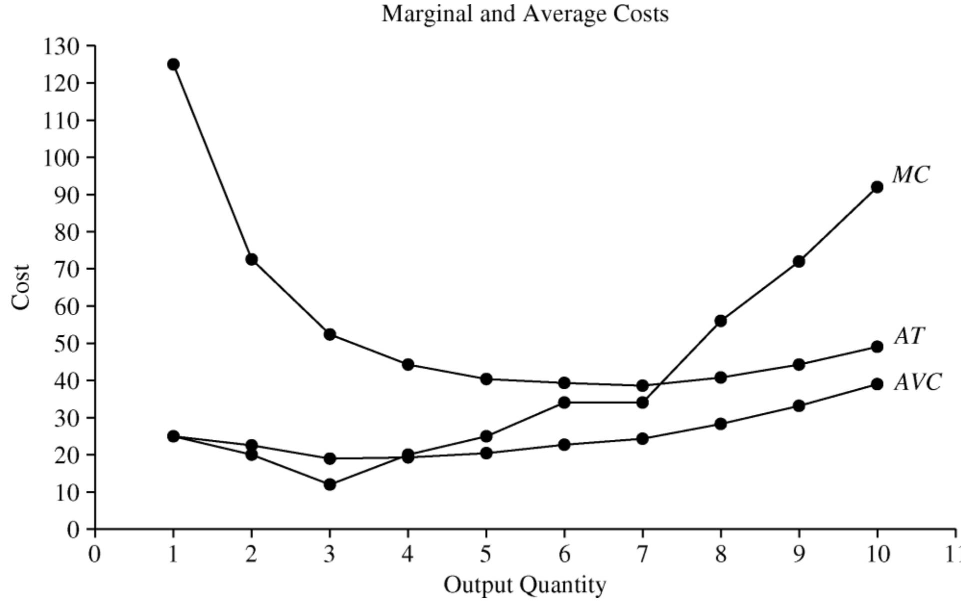
2. a. Fill in the blanks in the table on page 271 of the textbook.

Units of Output	Fixed Cost	Variable Cost	Total Cost	Marginal Cost	Average Fixed Cost	Average Variable Cost	Average Total Cost
0	100	0	100	—	—	—	—
1	100	25	125	25	100	25	125
2	100	45	145	20	50	22.50	72.50
3	100	57	157	12	33.33	19.00	52.33
4	100	77	177	20	25.00	19.25	44.25
5	100	102	202	25	20.00	20.40	40.40
6	100	136	236	34	16.67	22.67	39.33
7	100	170	270	34	14.29	24.29	38.57
8	100	226	326	56	12.50	28.25	40.75
9	100	298	398	72	11.11	33.11	44.22
10	100	390	490	92	10.00	39.00	49.00

b. Draw a graph that shows marginal cost, average variable cost, and average total cost, with cost on the vertical axis and quantity on the horizontal axis.

Average total cost is U-shaped and reaches a minimum at an output of about 7. Average variable cost is also U-shaped and reaches a minimum at an output between 3 and 4. Notice that average variable cost is always below average total cost. The difference between the two costs is the average fixed cost. Marginal cost is first diminishing, up to a quantity of 3, and then increases as q increases above 3. Marginal cost should intersect average variable cost and average total cost at their respective minimum points, though this is not accurately reflected in the table or the graph. If specific functions had been given in the problem instead of just a series of numbers, then it would be possible to find the exact point of intersection between marginal and average total cost and marginal and average variable cost. The curves are likely to intersect at a quantity that is not a whole number, and hence are not listed in the table or represented exactly in the cost diagram.

Marginal and Average Costs



3. A firm has a fixed production cost of $5000 and a constant marginal cost of production of $500 per unit produced.

a. What is the firm's total cost function? Average cost?

The variable cost of producing an additional unit, marginal cost, is constant at $500, so $VC = 500q$, and $AVC = \frac{VC}{q} = \frac{500q}{q} = 500$. Fixed cost is $5000 and therefore average fixed cost is $AFC = \frac{5000}{q}$. The total cost function is fixed cost plus variable cost or $TC = 5000 + 500q$.

Average total cost is the sum of average variable cost and average fixed cost: $ATC = 500 + \frac{5000}{q}$.

b. If the firm wanted to minimize the average total cost, would it choose to be very large or very small? Explain.

The firm would choose to be very large because average total cost decreases as q is increased. As q becomes extremely large, ATC will equal approximately 500 because the average fixed cost becomes close to zero.

4. Suppose a firm must pay an annual tax, which is a fixed sum, independent of whether it produces any output.

a. How does this tax affect the firm's fixed, marginal, and average costs?

This tax is a fixed cost because it does not vary with the quantity of output produced. If T is the amount of the tax and F is the firm's original fixed cost, the new total fixed cost increases to $TFC = T + F$. The tax does not affect marginal or variable cost because it does not vary with output. The tax increases both average fixed cost and average total cost by T/q.

b. Now suppose the firm is charged a tax that is proportional to the number of items it produces. Again, how does this tax affect the firm's fixed, marginal, and average costs?

Let t equal the per unit tax. When a tax is imposed on each unit produced, total variable cost increases by tq and fixed cost does not change. Average variable cost increases by t, and because fixed costs are constant, average total cost also increases by t. Further, because total cost increases by t for each additional unit produced, marginal cost increases by t.

5. A recent issue of *Business Week* reported the following:

During the recent auto sales slump, GM, Ford, and Chrysler decided it was cheaper to sell cars to rental companies at a loss than to lay off workers. That's because closing and reopening plants is expensive, partly because the auto makers' current union contracts obligate them to pay many workers even if they're not working.

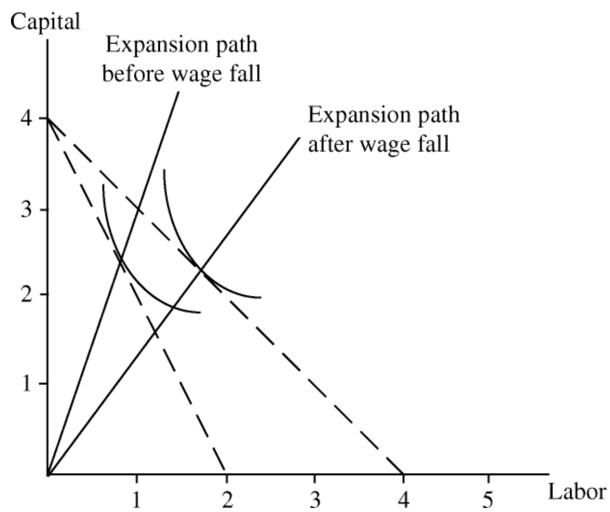
When the article discusses selling cars "at a loss," is it referring to accounting profit or economic profit? How will the two differ in this case? Explain briefly.

When the article refers to the car companies selling at a loss, it is referring to accounting profit. The article is stating that the price obtained for the sale of the cars to the rental companies was less than their accounting cost. Economic profit would be measured by the difference between the price and the opportunity cost of producing the cars. One major difference between accounting and economic cost in this case is the cost of labor. If the car companies must pay many workers even if they are not working, the wages paid to these workers are sunk. If the automakers have no alternative use for these workers (like doing repairs on the factory or preparing the companies' tax returns), the opportunity cost of using them to produce the rental cars is zero. Since the wages would be included in accounting costs, the accounting costs would be higher than the economic costs and would make the accounting profit lower than the economic profit.

6. **Suppose the economy takes a downturn, and that labor costs fall by 50% and are expected to stay at that level for a long time. Show graphically how this change in the relative price of labor and capital affects the firm's expansion path.**

The figure below shows a family of isoquants and two isocost curves. Units of capital are on the vertical axis and units of labor are on the horizontal axis. (***Note*:** The figure assumes that the production function underlying the isoquants implies linear expansion paths. However, the results do not depend on this assumption.)

If the price of labor decreases 50% while the price of capital remains constant, the isocost lines pivot outward. Because the expansion path is the set of points where the *MRTS* is equal to the ratio of prices, as the isocost lines become flatter, the expansion path becomes flatter and moves toward the labor axis. As a result the firm uses more labor relative to capital because labor has become less expensive.



7. **The cost of flying a passenger plane from point *A* to point *B* is $50,000. The airline flies this route four times per day at 7 AM, 10 AM, 1 PM, and 4 PM. The first and last flights are filled to capacity with 240 people. The second and third flights are only half full. Find the average cost per passenger for each flight. Suppose the airline hires you as a marketing consultant and wants to know which type of customer it should try to attract—the off-peak customer (the middle two flights) or the rush-hour customer (the first and last flights). What advice would you offer?**

The average cost per passenger is $50,000/240 = $208.33 for the full flights and $50,000/120 = $416.67 for the half full flights. The airline should focus on attracting more off-peak customers because there is excess capacity on the middle two flights. The marginal cost of taking another passenger on those two flights is almost zero, so the company will increase its profit if it can sell additional tickets for those flights, even if the ticket prices are less than average cost. The peak flights are already full, so attracting more customers at those times will not result in additional ticket sales.

8. You manage a plant that mass-produces engines by teams of workers using assembly machines. The technology is summarized by the production function $q = 5\ KL$, where q is the number of engines per week, K is the number of assembly machines, and L is the number of labor teams. Each assembly machine rents for r = $10,000 per week, and each team costs w = $5000 per week. Engine costs are given by the cost of labor teams and machines, plus $2000 per engine for raw materials. Your plant has a fixed installation of 5 assembly machines as part of its design.

a. What is the cost function for your plant—namely, how much would it cost to produce q engines? What are average and marginal costs for producing q engines? How do average costs vary with output?

The short-run production function is $q = 5(5)L = 25L$, because K is fixed at 5. Thus, for any level of output q, the number of labor teams hired will be $L = \frac{q}{25}$. The total cost function is thus given by the sum of the costs of capital, labor, and raw materials:

$$TC(q) = rK + wL + 2000q = (10{,}000)(5) + (5{,}000)\left(\frac{q}{25}\right) + 2000q$$

$$TC(q) = 50{,}000 + 2200q.$$

The average cost function is then given by:

$$AC(q) = \frac{TC(q)}{q} = \frac{50{,}000 + 2200q}{q}.$$

and the marginal cost function is given by:

$$MC(q) = \frac{dTC}{dq} = 2200.$$

Marginal costs are constant at $2200 per engine and average costs will decrease as quantity increases because the average fixed cost of capital decreases.

b. How many teams are required to produce 250 engines? What is the average cost per engine?

To produce q = 250 engines we need $L = \frac{q}{25}$, so L = 10 labor teams. Average costs are $2400 as shown below:

$$AC(q = 250) = \frac{50{,}000 + 2200(250)}{250} = 2400.$$

c. You are asked to make recommendations for the design of a new production facility. What capital/labor (K/L) ratio should the new plant accommodate if it wants to minimize the total cost of producing at any level of output q?

We no longer assume that K is fixed at 5. We need to find the combination of K and L that minimizes cost at any level of output q. The cost-minimization rule is given by

$$\frac{MP_K}{r} = \frac{MP_L}{w}.$$

To find the marginal product of capital, observe that increasing K by 1 unit increases q by $5L$, so $MP_K = 5L$. Similarly, observe that increasing L by 1 unit increases q by $5K$, so $MP_L = 5K$. Mathematically,

$$MP_K = \frac{\partial q}{\partial K} = 5L \text{ and } MP_L = \frac{\partial q}{\partial L} = 5K.$$

Using these formulas in the cost-minimization rule, we obtain:

$$\frac{5L}{r} = \frac{5K}{w} \Rightarrow \frac{K}{L} = \frac{w}{r} = \frac{5000}{10{,}000} = \frac{1}{2}.$$

The new plant should accommodate a capital to labor ratio of 1 to 2, and this is the same regardless of the number of units produced.

9. The short-run cost function of a company is given by the equation $TC = 200 + 55q$, where TC is the total cost and q is the total quantity of output, both measured in thousands.

a. What is the company's fixed cost?

When $q = 0$, $TC = 200$, so fixed cost is equal to 200 (or $200,000).

b. If the company produced 100,000 units of goods, what would be its average variable cost?

With 100,000 units, $q = 100$. Variable cost is $55q = (55)(100) = 5500$ (or $5,500,000). Average variable cost is $\frac{VC}{q} = \frac{5500}{100} = 55$ (or $55,000).

c. What would be its marginal cost of production?

With constant average variable cost, marginal cost is equal to average variable cost, which is 55 (or $55,000).

d. What would be its average fixed cost?

At $q = 100$, average fixed cost is $\frac{FC}{q} = \frac{200}{100} = 2$ (or $2000).

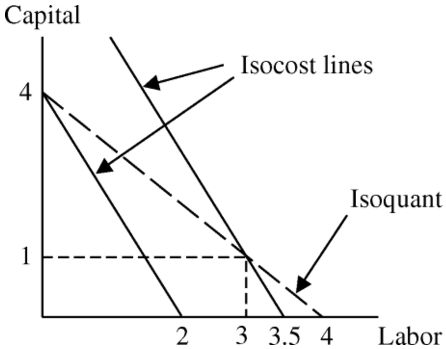
e. Suppose the company borrows money and expands its factory. Its fixed cost rises by $50,000, but its variable cost falls to $45,000 per 1000 units. The cost of interest (i) also enters into the equation. Each 1-point increase in the interest rate raises costs by $3000. Write the new cost equation.

Fixed cost changes from 200 to 250, measured in thousands. Variable cost decreases from 55 to 45, also measured in thousands. Fixed cost also includes interest charges of $3i$. The cost equation is $TC = 250 + 45q + 3i$.

10. A chair manufacturer hires its assembly-line labor for $30 an hour and calculates that the rental cost of its machinery is $15 per hour. Suppose that a chair can be produced using 4 hours of labor or machinery in any combination. If the firm is currently using 3 hours of labor for each hour of machine time, is it minimizing its costs of production? If so, why? If not, how can it improve the situation? Graphically illustrate the isoquant and the two isocost lines for the current combination of labor and capital and for the optimal combination of labor and capital.

If the firm can produce one chair with either four hours of labor or four hours of machinery (i.e., capital), or any combination, then the isoquant is a straight line with a slope of −1 and intercepts at $K = 4$ and $L = 4$, as depicted by the dashed line.

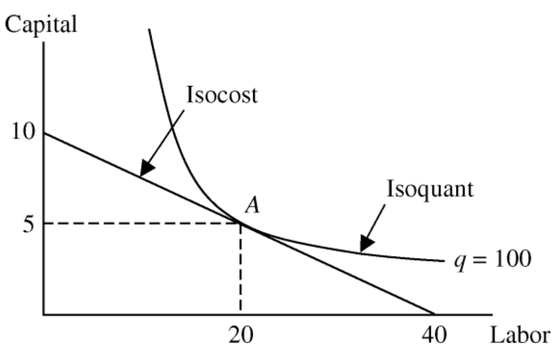
The isocost lines, $TC = 30L + 15K$, have slopes of $-30/15 = -2$ when plotted with capital on the vertical axis and intercepts at $K = TC/15$ and $L = TC/30$. The cost minimizing point is the corner solution where $L = 0$ and $K = 4$, so the firm is not currently minimizing its costs. At the optimal point, total cost is \$60. Two isocost lines are illustrated on the graph. The first one is further from the origin and represents the current higher cost (\$105) of using 3 labor and 1 capital. The firm will find it optimal to move to the second isocost line which is closer to the origin, and which represents a lower cost (\$60). In general, the firm wants to be on the lowest isocost line possible, which is the lowest isocost line that still intersects or touches the given isoquant.



11. Suppose that a firm's production function is $q = 10L^{\frac{1}{2}}K^{\frac{1}{2}}$. The cost of a unit of labor is \$20 and the cost of a unit of capital is \$80.

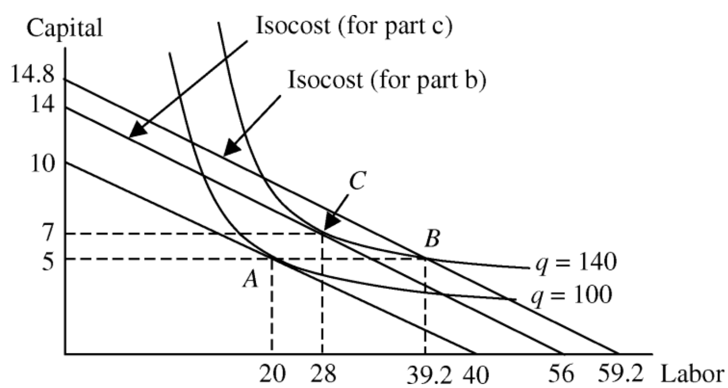
a. The firm is currently producing 100 units of output and has determined that the cost-minimizing quantities of labor and capital are 20 and 5, respectively. Graphically illustrate this using isoquants and isocost lines.

To graph the isoquant, set $q = 100$ in the production function and solve it for K. Solving for K: $K^{1/2} = \dfrac{q}{10L^{1/2}}$ Substitute 100 for q and square both sides. The isoquant is $K = 100/L$. Choose various combinations of L and K and plot them. The isoquant is convex. The optimal quantities of labor and capital are given by the point where the isocost line is tangent to the isoquant. The isocost line has a slope of $-1/4$, given labor is on the horizontal axis. The total cost is $TC = (\$20)(20) + (\$80)(5) = \$800$, so the isocost line has the equation $20L + 80K = 800$, or $K = 10 - 0.25L$, with intercepts $K = 10$ and $L = 40$. The optimal point is labeled A on the graph.



b. The firm now wants to increase output to 140 units. If capital is fixed in the short run, how much labor will the firm require? Illustrate this graphically and find the firm's new total cost.

The new level of labor is 39.2. To find this, use the production function $q = 10L^{\frac{1}{2}}K^{\frac{1}{2}}$ and substitute 140 for output and 5 for capital; then solve for L. The new cost is TC = (\$20)(39.2) + (\$80)(5) = \$1184. The new isoquant for an output of 140 is above and to the right of the original isoquant. Since capital is fixed in the short run, the firm will move out horizontally to the new isoquant and new level of labor. This is point B on the graph below. This is not the long-run cost-minimizing point, but it is the best the firm can do in the short run with K fixed at 5. You can tell that this is not the long-run optimum because the isocost is not tangent to the isoquant at point B. Also there are points on the new (q = 140) isoquant that are below the new isocost (for part b) line. These points all involve hiring more capital and less labor.



c. Graphically identify the cost-minimizing level of capital and labor in the long run if the firm wants to produce 140 units.

This is point C on the graph above. When the firm is at point B it is not minimizing cost. The firm will find it optimal to hire more capital and less labor and move to the new lower isocost (for part c) line that is tangent to the q = 140 isoquant. Note that all three isocost lines are parallel and have the same slope.

d. If the marginal rate of technical substitution is $\frac{K}{L}$, find the optimal level of capital and labor required to produce the 140 units of output.

Set the marginal rate of technical substitution equal to the ratio of the input costs so that $\frac{K}{L} = \frac{20}{80} \Rightarrow K = \frac{L}{4}$. Now substitute this into the production function for K, set q equal to 140, and solve for L: $140 = 10L^{\frac{1}{2}}\left(\frac{L}{4}\right)^{\frac{1}{2}} \Rightarrow L = 28,\ K = 7.$ This is point C on the graph. The new cost is TC = (\$20)(28) + (\$80)(7) = \$1120, which is less than in the short run (part b), because the firm can adjust all its inputs in the long run.

12. A computer company's cost function, which relates its average cost of production AC to its cumulative output in thousands of computers Q and its plant size in terms of thousands of computers produced per year q (within the production range of 10,000 to 50,000 computers), is given by

$$AC = 10 - 0.1Q + 0.3q.$$

a. Is there a learning curve effect?

The learning curve describes the relationship between the cumulative output and the inputs required to produce a unit of output. Average cost measures the input requirements per unit of output. Learning curve effects exist if average cost falls with increases in cumulative output. Here, average cost decreases by \$0.10 each time cumulative output Q increases by 1000. Therefore, there are learning curve effects.

b. Are there economies or diseconomies of scale?

There are diseconomies of scale. Holding cumulative output, Q, constant, there are diseconomies of scale if the average cost increases as annual output q increases. In this example, average cost increases by \$0.30 for each additional one thousand computers produced, so there are diseconomies of scale.

c. During its existence, the firm has produced a total of 40,000 computers and is producing 10,000 computers this year. Next year it plans to increase production to 12,000 computers. Will its average cost of production increase or decrease? Explain.

First, calculate average cost this year:

$$AC_1 = 10 - 0.1Q + 0.3q = 10 - (0.1)(40) + (0.3)(10) = 9.00.$$

Second, calculate the average cost next year:

$$AC_2 = 10 - (0.1)(50) + (0.3)(12) = 8.60.$$

(***Note*:** Cumulative output has increased from 40,000 to 50,000, hence $Q = 50$ next year.) The average cost will decrease because of the learning effect, and despite the diseconomies of scale involved when annual output increases from 10 to 12 thousand computers.

13. Suppose the long-run total cost function for an industry is given by the cubic equation $TC = a + bq + cq^2 + dq^3$. Show (using calculus) that this total cost function is consistent with a U-shaped average cost curve for at least some values of a, b, c, and d.

To show that the cubic cost equation implies a U-shaped average cost curve, we use algebra, calculus, and economic reasoning to place sign restrictions on the parameters of the equation. These techniques are illustrated by the example below.

First, if output is equal to zero, then $TC = a$, where a represents fixed costs. In the short run, fixed costs are positive, $a > 0$, but in the long run, where all inputs are variable $a = 0$. Therefore, we restrict a to be zero.

Next, we know that average cost must be positive. Dividing TC by q, with $a = 0$:

$$AC = b + cq + dq^2.$$

This equation is simply a quadratic function. When graphed, it has two basic shapes: a U shape and a hill (upside down U) shape. We want the U, i.e., a curve with a minimum (minimum average cost), rather than a hill with a maximum.

At the minimum, the slope should be zero, thus the first derivative of the average cost curve with respect to q must be equal to zero. For a U-shaped AC curve, the second derivative of the average cost curve must be positive.

The first derivative is $c + 2dq$; the second derivative is $2d$. If the second derivative is to be positive, then $d > 0$. If the first derivative is to equal zero, then solving for c as a function of q and d yields: $c = -2dq$. Since d is positive, and the minimum AC must be at some point where q is positive, then c must be negative: $c < 0$.

To restrict b, we know that at its minimum, average cost must be positive. The minimum occurs when $c + 2dq = 0$. Solve for q as a function of c and d: $q = -c/2d > 0$. Next, substituting this value for q into the expression for average cost, and simplifying the equation:

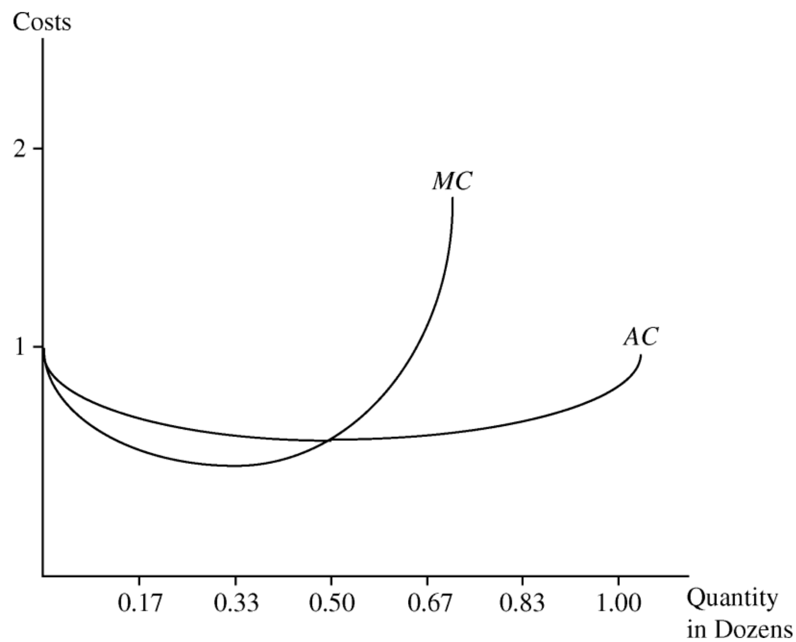
$$AC = b + cq + dq^2 = b + c\left(\frac{-c}{2d}\right) + d\left(\frac{-c}{2d}\right)^2, \text{ or}$$

$$AC = b - \frac{c^2}{2d} + \frac{c^2}{4d} = b - \frac{2c^2}{4d} + \frac{c^2}{4d} = b - \frac{c^2}{4d} > 0.$$

This implies $b > \dfrac{c^2}{4d}$. Because $c^2 > 0$ and $d > 0$, b must be positive.

In summary, for U-shaped long-run average cost curves, a must be zero, b and d must be positive, c must be negative, and $4db > c^2$. However, these conditions do not ensure that marginal cost is positive. To insure that marginal cost has a U shape and that its minimum is positive, use the same procedure, i.e., solve for q at minimum marginal cost: $q = -c/3d$. Then substitute into the expression for marginal cost: $b + 2cq + 3dq^2$. From this we find that c^2 must be less than $3bd$. Notice that parameter values that satisfy this condition also satisfy $4db > c^2$, but not the reverse, so $c^2 < 3bd$ is the more stringent requirement.

For example, let $a = 0$, $b = 1$, $c = -1$, $d = 1$. These values satisfy all the restrictions derived above. Total cost is $q - q^2 + q^3$, average cost is $1 - q + q^2$, and marginal cost is $1 - 2q + 3q^2$. Minimum average cost is where $q = 1/2$ and minimum marginal cost is where $q = 1/3$ (think of q as dozens of units, so no fractional units are produced). See the figure below.



14. A computer company produces hardware and software using the same plant and labor. The total cost of producing computer processing units *H* and software programs *S* is given by

$$TC = aH + bS - cHS$$

where *a*, *b*, and *c* are positive. Is this total cost function consistent with the presence of economies or diseconomies of scale? With economies or diseconomies of scope?

If each product were produced by itself there would be neither economies nor diseconomies of scale. To see this, define the total cost of producing H alone (TC_H) to be the total cost when $S = 0$. Thus $TC_H = aH$. Similarly, $TC_S = bS$. In both cases, doubling the number of units produced doubles the total cost, so there are no economies or diseconomies of scale.

Economies of scope exist if $S_C > 0$, where, from equation (7.7) in the text:

$$SC = \frac{C(q_1) + C(q_2) - C(q_1, q_2)}{C(q_1, q_2)}.$$

In our case, $C(q_1)$ is TC_H, $C(q_2)$ is TC_S, and $C(q_1, q_2)$ is TC. Therefore,

$$SC = \frac{aH + bS - (aH + bS - cHS)}{aH + bS - cHS} = \frac{cHS}{aH + bS - cHS}.$$

Because cHS (the numerator) and TC (the denominator) are both positive, it follows that $SC > 0$, and hence there are economies of scope.