

Chapter 12

Monopolistic Competition and Oligopoly

■ Review Questions

1. **What are the characteristics of a monopolistically competitive market? What happens to the equilibrium price and quantity in such a market if one firm introduces a new, improved product?**

The two primary characteristics of a monopolistically competitive market are that (1) firms compete by selling differentiated products that are highly, but not perfectly, substitutable and (2) there is free entry and exit from the market. When a new firm enters a monopolistically competitive market or one firm introduces an improved product, the demand curve for each of the other firms shifts inward, reducing the price and quantity received by those incumbents. Thus, the introduction of a new product by a firm will reduce the price received and quantity sold of existing products.

2. **Why is the firm's demand curve flatter than the total market demand curve in monopolistic competition? Suppose a monopolistically competitive firm is making a profit in the short run. What will happen to its demand curve in the long run?**

The flatness or steepness of the firm's demand curve is a function of the elasticity of demand for the firm's product. The elasticity of the firm's demand curve is greater than the elasticity of market demand because it is easier for consumers to switch to another firm's highly substitutable product than to switch consumption to an entirely different product. Profit in the short run induces other firms to enter. As new firms enter, the incumbent firm's demand and marginal revenue curves shift to the left, reducing the profit-maximizing quantity. In the long run profits fall to zero, leaving no incentive for more firms to enter.

3. **Some experts have argued that too many brands of breakfast cereal are on the market. Give an argument to support this view. Give an argument against it.**

Pro: Too many brands of any single product signals excess capacity, implying that each firm is producing an output level smaller than the level that would minimize average cost. Limiting the number of brands would therefore enhance overall economic efficiency.

Con: Consumers value the freedom to choose among a wide variety of competing products. Even if costs are slightly higher as a result of the large number of brands available, the benefits to consumers outweigh the extra costs.

(*Note:* In 1972 the Federal Trade Commission filed suit against Kellogg, General Mills, and General Foods. It charged that these firms attempted to suppress entry into the cereal market by introducing 150 heavily advertised brands between 1950 and 1970, crowding competitors off grocers' shelves. This case was eventually dismissed in 1982.)

4. **Why is the Cournot equilibrium stable? (i.e., Why don't firms have any incentive to change their output levels once in equilibrium?) Even if they can't collude, why don't firms set their**

outputs at the joint profit-maximizing levels (i.e., the levels they would have chosen had they colluded)?

A Cournot equilibrium is stable because each firm is producing the amount that maximizes its profit, *given what its competitors are producing*. If all firms behave this way, no firm has an incentive to change its output. Without collusion, firms find it difficult to agree tacitly to reduce output.

If all firms were producing at the joint profit-maximizing level, each would have an incentive to increase output, because that would increase each firm's profit at the expense of the firms that were limiting sales. But when each firm increases output, they all end up back at the Cournot equilibrium. Thus it is very difficult to reach the joint profit-maximizing level without overt collusion, and even then it may be difficult to prevent cheating among the cartel members.

5. In the Stackelberg model, the firm that sets output first has an advantage. Explain why.

The Stackelberg leader gains an advantage because the second firm must accept the leader's large output as given and produce a smaller output for itself. If the second firm decided to produce a larger quantity, this would reduce price and profit for both firms. The first firm knows that the second firm will have no choice but to produce a smaller output in order to maximize profit, and thus the first firm is able to capture a larger share of industry profits.

6. What do the Cournot and Bertrand models have in common? What is different about the two models?

Both are oligopoly models in which firms produce a homogeneous good. In the Cournot model, each firm assumes its rivals will not change the quantity they produce. In the Bertrand model, each firm assumes its rivals will not change the price they charge. In both models, each firm takes some aspect of its rivals' behavior (either quantity or price) as fixed when making its own decision. The difference between the two is that in the Bertrand model firms end up producing where price equals marginal cost, whereas in the Cournot model the firms will produce more than the monopoly output but less than the competitive output.

7. Explain the meaning of a Nash equilibrium when firms are competing with respect to price. Why is the equilibrium stable? Why don't the firms raise prices to the level that maximizes joint profits?

A Nash equilibrium in price competition occurs when each firm chooses its price, assuming *its competitors' prices will not change*. In equilibrium, each firm is doing the best it can, conditional on its competitors' prices. The equilibrium is stable because each firm is maximizing its profit, and therefore no firm has an incentive to raise or lower its price.

No individual firm would raise its price to the level that maximizes joint profit if the other firms do not do the same, because it would lose sales to the firms with lower prices. It is also difficult to collude. A cartel agreement is difficult to enforce because each firm has an incentive to cheat. By lowering price, the cheating firm can increase its market share and profits. A second reason that firms do not collude is that such behavior violates antitrust laws. In particular, price fixing violates Section 1 of the Sherman Act. Of course, there are attempts to circumvent antitrust laws through tacit collusion.

8. The kinked demand curve describes price rigidity. Explain how the model works. What are its limitations? Why does price rigidity occur in oligopolistic markets?

According to the kinked demand curve model, each firm faces a demand curve that is kinked at the currently prevailing price. Each firm believes that if it raises its price, the other firms will not raise their prices, and thus many of the firm's customers will shift their purchases to competitors. This reasoning implies a highly elastic demand for price increases. On the other hand, each firm believes that if it lowers its price, its competitors will also lower their prices, and the firm will not increase sales by much. This implies a demand curve that is less elastic for price decreases than for price increases. This kink in the demand curve leads to a discontinuity in the marginal revenue curve, so only large changes in marginal cost lead to changes in price.

A major limitation is that the kinked-demand model does not explain *how* the starting price is determined. Price rigidity may occur in oligopolistic markets because firms want to avoid destructive price wars. Managers learn from experience that cutting prices does not lead to lasting increases in profits. As a result, firms are reluctant to "rock the boat" by changing prices even when costs change.

9. Why does price leadership sometimes evolve in oligopolistic markets? Explain how the price leader determines a profit-maximizing price.

Since firms cannot explicitly coordinate on setting price, they use implicit means. One form of implicit collusion is to follow a price leader. The price leader, often the largest or dominant firm in the industry, determines its profit-maximizing quantity by calculating the demand curve it faces as follows: at each price, it subtracts the quantity supplied by all other firms from the market demand, and the residual is its demand curve. The leader chooses the quantity that equates its marginal revenue with its marginal cost and sets price to sell this quantity. The other firms (the followers) match the leader's price and supply the remainder of the market.

10. Why has the OPEC oil cartel succeeded in raising prices substantially while the CIPEC copper cartel has not? What conditions are necessary for successful cartelization? What organizational problems must a cartel overcome?

Successful cartelization requires two characteristics: demand should be relatively inelastic, and the cartel must be able to control most of the supply. OPEC succeeded in the short run because the short-run demand and supply of oil were both inelastic. CIPEC has not been successful because both demand and non-CIPEC supply were highly responsive to price. A cartel faces two organizational problems: agreement on a price and a division of the market among cartel members, and it must monitor and enforce the agreement.

■ Exercises

1. Suppose all firms in a monopolistically competitive industry were merged into one large firm. Would that new firm produce as many different brands? Would it produce only a single brand? Explain.

Monopolistic competition is defined by product differentiation. Each firm earns economic profit by distinguishing its brand from all other brands. This distinction can arise from underlying differences in the product or from differences in advertising. If these competitors merge into a single firm, the resulting monopolist would not produce as many brands, since too much brand competition is internecine (mutually destructive). However, it is unlikely that only one brand would be produced after the merger. Producing several brands with different prices and characteristics is one method of splitting the market into sets of customers with different tastes and price elasticities. The monopolist

can sell to more consumers and maximize overall profit by producing multiple brands and practicing a form of price discrimination.

2. Consider two firms facing the demand curve $P = 50 - 5Q$, where $Q = Q_1 + Q_2$. The firms' cost functions are $C_1(Q_1) = 20 + 10Q_1$ and $C_2(Q_2) = 10 + 12Q_2$.
- a. Suppose both firms have entered the industry. What is the joint profit-maximizing level of output? How much will each firm produce? How would your answer change if the firms have not yet entered the industry?

If the firms collude, they face the market demand curve, so their marginal revenue curve is:

$$MR = 50 - 10Q.$$

Set marginal revenue equal to marginal cost (the marginal cost of Firm 1, since it is lower than that of Firm 2) to determine the profit-maximizing quantity, Q :

$$50 - 10Q = 10, \text{ or } Q = 4.$$

Substituting $Q = 4$ into the demand function to determine price:

$$P = 50 - 5(4) = \$30.$$

The question now is how the firms will divide the total output of 4 among themselves. The joint profit-maximizing solution is for Firm 1 to produce all of the output because its marginal cost is less than Firm 2's marginal cost. We can ignore fixed costs because both firms are already in the market and will be saddled with their fixed costs no matter how many units each produces. If Firm 1 produces all 4 units, its profit will be

$$\pi_1 = (30)(4) - (20 + (10)(4)) = \$60.$$

The profit for Firm 2 will be:

$$\pi_2 = (30)(0) - (10 + (12)(0)) = -\$10.$$

Total industry profit will be:

$$\pi_T = \pi_1 + \pi_2 = 60 - 10 = \$50.$$

Firm 2, of course, will not like this. One solution is for Firm 1 to pay Firm 2 \$35 so that both earn a profit of \$25, although they may well disagree about the amount to be paid. If they split the output evenly between them, so that each firm produces 2 units, then total profit would be \$46 (\$20 for Firm 1 and \$26 for Firm 2). This does not maximize total profit, but Firm 2 would prefer it to the \$25 it gets from an even split of the maximum \$50 profit. So there is no clear-cut answer to this question.

If Firm 1 were the only entrant, its profits would be \$60 and Firm 2's would be 0.

If Firm 2 were the only entrant, then it would equate marginal revenue with its marginal cost to determine its profit-maximizing quantity:

$$50 - 10Q_2 = 12, \text{ or } Q_2 = 3.8.$$

Substituting Q_2 into the demand equation to determine price:

$$P = 50 - 5(3.8) = \$31.$$

The profits for Firm 2 would be:

$$\pi_2 = (31)(3.8) - (10 + (12)(3.8)) = \$62.20,$$

and Firm 1 would earn 0. Thus, Firm 2 would make a larger profit than Firm 1 if it were the only firm in the market, because Firm 2 has lower fixed costs.

- b. What is each firm's equilibrium output and profit if they behave noncooperatively? Use the Cournot model. Draw the firms' reaction curves and show the equilibrium.**

In the Cournot model, Firm 1 takes Firm 2's output as given and maximizes profits. Firm 1's profit function is

$$\begin{aligned}\pi_1 &= (50 - 5Q_1 - 5Q_2)Q_1 - (20 + 10Q_1), \text{ or} \\ \pi_1 &= 40Q_1 - 5Q_1^2 - 5Q_1Q_2 - 20.\end{aligned}$$

Setting the derivative of the profit function with respect to Q_1 to zero, we find Firm 1's reaction function:

$$\frac{\partial \pi_1}{\partial Q_1} = 40 - 10Q_1 - 5Q_2 = 0, \text{ or } Q_1 = 4 - \left(\frac{Q_2}{2}\right).$$

Similarly, Firm 2's reaction function is $Q_2 = 3.8 - \left(\frac{Q_1}{2}\right)$.

To find the Cournot equilibrium, substitute Firm 2's reaction function into Firm 1's reaction function:

$$Q_1 = 4 - \left(\frac{1}{2}\right)\left(3.8 - \frac{Q_1}{2}\right), \text{ or } Q_1 = 2.8.$$

Substituting this value for Q_1 into the reaction function for Firm 2, we find

$$Q_2 = 2.4.$$

Substituting the values for Q_1 and Q_2 into the demand function to determine the equilibrium price:

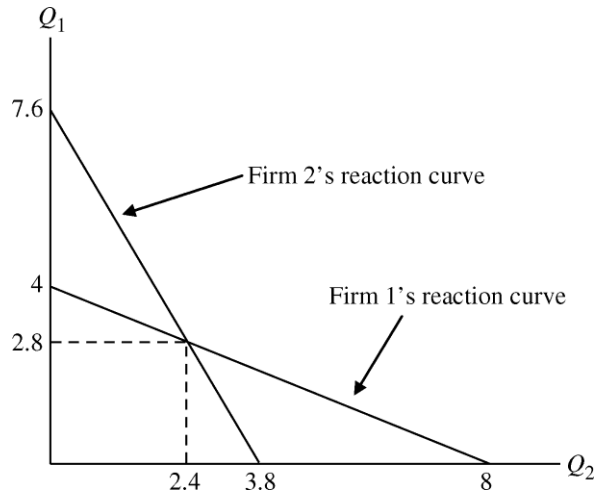
$$P = 50 - 5(2.8 + 2.4) = \$24.$$

The profits for Firms 1 and 2 are equal to

$$\pi_1 = (24)(2.8) - (20 + (10)(2.8)) = \$19.20, \text{ and}$$

$$\pi_2 = (24)(2.4) - (10 + (12)(2.4)) = \$18.80.$$

The firms' reaction curves and the Cournot equilibrium are shown below.



- c. **How much should Firm 1 be willing to pay to purchase Firm 2 if collusion is illegal but a takeover is not?**

To determine how much Firm 1 will be willing to pay to purchase Firm 2, we must compare Firm 1's profits in the monopoly situation versus its profits in an oligopoly. The difference between the two will be what Firm 1 is willing to pay for Firm 2.

From part a, Firm 1's profit when it sets marginal revenue equal to its marginal cost is \$60. This is what the firm would earn if it was a monopolist. From part b, profit is \$19.20 for Firm 1 when the firms compete against each other in a Cournot-type market. Firm 1 should therefore be willing to pay up to $\$60 - 19.20 = \40.80 for Firm 2.

3. **A monopolist can produce at a constant average (and marginal) cost of $AC = MC = \$5$. It faces a market demand curve given by $Q = 53 - P$.**

- a. **Calculate the profit-maximizing price and quantity for this monopolist. Also calculate its profits.**

First solve for the inverse demand curve, $P = 53 - Q$. Then the marginal revenue curve has the same intercept and twice the slope:

$$MR = 53 - 2Q.$$

Marginal cost is a constant \$5. Setting $MR = MC$, find the optimal quantity:

$$53 - 2Q = 5, \text{ or } Q = 24.$$

Substitute $Q = 24$ into the demand function to find price:

$$P = 53 - 24 = \$29.$$

Assuming fixed costs are zero, profits are equal to

$$\pi = TR - TC = (29)(24) - (5)(24) = \$576.$$

- b. **Suppose a second firm enters the market. Let Q_1 be the output of the first firm and Q_2 be the output of the second. Market demand is now given by**

$$Q_1 + Q_2 = 53 - P.$$

Assuming that this second firm has the same costs as the first, write the profits of each firm as functions of Q_1 and Q_2 .

When the second firm enters, price can be written as a function of the output of both firms: $P = 53 - Q_1 - Q_2$. We may write the profit functions for the two firms:

$$\pi_1 = PQ_1 - C(Q_1) = (53 - Q_1 - Q_2)Q_1 - 5Q_1, \text{ or } \pi_1 = 48Q_1 - Q_1^2 - Q_1Q_2$$

and

$$\pi_2 = PQ_2 - C(Q_2) = (53 - Q_1 - Q_2)Q_2 - 5Q_2, \text{ or } \pi_2 = 48Q_2 - Q_2^2 - Q_1Q_2.$$

- c. **Suppose (as in the Cournot model) that each firm chooses its profit-maximizing level of output on the assumption that its competitor's output is fixed. Find each firm's "reaction curve" (i.e., the rule that gives its desired output in terms of its competitor's output).**

Under the Cournot assumption, each firm treats the output of the other firm as a constant in its maximization calculations. Therefore, Firm 1 chooses Q_1 to maximize π_1 in part b with Q_2 being treated as a constant. The change in π_1 with respect to a change in Q_1 is

$$\frac{\partial \pi_1}{\partial Q_1} = 48 - 2Q_1 - Q_2 = 0, \text{ or } Q_1 = 24 - \frac{Q_2}{2}.$$

This equation is the reaction function for Firm 1, which generates the profit-maximizing level of output, given the output of Firm 2. Because the problem is symmetric, the reaction function for Firm 2 is

$$Q_2 = 24 - \frac{Q_1}{2}.$$

- d. **Calculate the Cournot equilibrium (i.e., the values of Q_1 and Q_2 for which each firm is doing as well as it can given its competitor's output). What are the resulting market price and profits of each firm?**

Solve for the values of Q_1 and Q_2 that satisfy both reaction functions by substituting Firm 2's reaction function into the function for Firm 1:

$$Q_1 = 24 - \left(\frac{1}{2}\right)\left(24 - \frac{Q_1}{2}\right), \text{ or } Q_1 = 16.$$

By symmetry, $Q_2 = 16$.

To determine the price, substitute Q_1 and Q_2 into the demand equation:

$$P = 53 - 16 - 16 = \$21.$$

Profit for Firm 1 is therefore

$$\pi_i = PQ_i - C(Q_i) = \pi_i = (21)(16) - (5)(16) = \$256.$$

Firm 2's profit is the same, so total industry profit is $\pi_1 + \pi_2 = \$256 + \$256 = \$512$.

- e. **Suppose there are N firms in the industry, all with the same constant marginal cost, $MC = \$5$. Find the Cournot equilibrium. How much will each firm produce, what will be the market price, and how much profit will each firm earn? Also, show that as N becomes large, the market price approaches the price that would prevail under perfect competition.**

If there are N identical firms, then the price in the market will be

$$P = 53 - (Q_1 + Q_2 + \dots)$$

Profits for the i th firm are given by

$$\begin{aligned} \pi_i &= PQ_i - C(Q_i), \\ \pi_i &= 53Q_i - Q_1Q_i - Q_2Q_i - \dots \quad \dots \quad 5Q_i. \end{aligned}$$

Differentiating to obtain the necessary first-order condition for profit maximization,

$$\frac{\partial \pi_i}{\partial Q_i} = 53 - Q_1 - Q_2 - \dots \quad \dots \quad = 0.$$

Solving for Q_i ,

$$Q_i = 24 - \frac{1}{2}(Q_1 + \dots \quad \dots)$$

If all firms face the same costs, they will all produce the same level of output, i.e., $Q_i = Q^*$. Therefore,

$$\begin{aligned} Q^* &= 24 - \frac{1}{2}(N-1)Q^*, \text{ or } 2Q^* = 48 - (N-1)Q^*, \text{ or} \\ (N+1)Q^* &= 48, \text{ or } Q^* = \frac{48}{(N+1)}. \end{aligned}$$

Now substitute $Q = NQ^*$ for total output in the demand function:

$$P = 53 - N\left(\frac{48}{N+1}\right).$$

Total profits are

$$\pi_T = PQ - C(Q) = P(NQ^*) - 5(NQ^*)$$

or

$$\begin{aligned} \pi_T &= \left[53 - N\left(\frac{48}{N+1}\right)\right](N)\left(\frac{48}{N+1}\right) - 5N\left(\frac{48}{N+1}\right) \text{ or} \\ \pi_T &= \left[48 - (N)\left(\frac{48}{N+1}\right)\right](N)\left(\frac{48}{N+1}\right) \end{aligned}$$

or

$$\pi_T = (48)\left(\frac{N+1-N}{N+1}\right)(48)\left(\frac{N}{N+1}\right) = (2,304)\left(\frac{N}{(N+1)}\right)$$

Notice that with N firms

$$Q = 48\left(\frac{N}{N+1}\right)$$

and that, as N increases ($N \rightarrow \infty$)

$$Q = 48.$$

Similarly, with

$$P = 53 - 48\left(\frac{N}{N+1}\right),$$

as $N \rightarrow \infty$,

$$P = 53 - 48 = 5.$$

Finally,

$$\pi_T = 2,304 \left(\frac{N}{(N+1)^2} \right),$$

so as $N \rightarrow \infty$,

$$\pi_T = \$0.$$

In perfect competition, we know that profits are zero and price equals marginal cost. Here, $\pi_T = \$0$ and $P = MC = 5$. Thus, when N approaches infinity, this market approaches a perfectly competitive one.

4. This exercise is a continuation of Exercise 3. We return to two firms with the same constant average and marginal cost, $AC = MC = 5$, facing the market demand curve $Q_1 + Q_2 = 53 - P$. Now we will use the Stackelberg model to analyze what will happen if one of the firms makes its output decision before the other.

a. Suppose Firm 1 is the Stackelberg leader (i.e., makes its output decisions before Firm 2). Find the reaction curves that tell each firm how much to produce in terms of the output of its competitor.

Firm 2's reaction curve is the same as determined in part c of Exercise 3:

$$Q_2 = 24 - \left(\frac{Q_1}{2} \right).$$

Firm 1 does not have a reaction function because it makes its output decision before Firm 2, so there is nothing to react to. Instead, Firm 1 uses its knowledge of Firm 2's reaction function when determining its optimal output as shown in part b below.

b. How much will each firm produce, and what will its profit be?

Firm 1, the Stackelberg leader, will choose its output, Q_1 , to maximize its profits, subject to the reaction function of Firm 2:

$$\max \pi_1 = PQ_1 - C(Q_1),$$

subject to

$$Q_2 = 24 - \left(\frac{Q_1}{2} \right).$$

Substitute for Q_2 in the demand function and, after solving for P , substitute for P in the profit function:

$$\max \pi_1 = \left(53 - Q_1 - \left(24 - \frac{Q_1}{2} \right) \right) (Q_1) - 5Q_1.$$

To determine the profit-maximizing quantity, we find the change in the profit function with respect to a change in Q_1 :

$$\frac{d\pi_1}{dQ_1} = 53 - 2Q_1 - 24 + Q_1 - 5.$$

Set this expression equal to 0 to determine the profit-maximizing quantity:

$$53 - 2Q_1 - 24 + Q_1 - 5 = 0, \text{ or } Q_1 = 24.$$

Substituting $Q_1 = 24$ into Firm 2's reaction function gives Q_2 :

$$Q_2 = 24 - \frac{24}{2} = 12.$$

Substitute Q_1 and Q_2 into the demand equation to find the price:

$$P = 53 - 24 - 12 = \$17.$$

Profits for each firm are equal to total revenue minus total costs, or

$$\pi_1 = (17)(24) - (5)(24) = \$288, \text{ and}$$

$$\pi_2 = (17)(12) - (5)(12) = \$144.$$

Total industry profit, $\pi_T = \pi_1 + \pi_2 = \$288 + \$144 = \432 .

Compared to the Cournot equilibrium, total output has increased from 32 to 36, price has fallen from \$21 to \$17, and total profits have fallen from \$512 to \$432. Profits for Firm 1 have risen from \$256 to \$288, while the profits of Firm 2 have declined sharply from \$256 to \$144.

5. Two firms compete in selling identical widgets. They choose their output levels Q_1 and Q_2 simultaneously and face the demand curve

$$P = 30 - Q$$

where $Q = Q_1 + Q_2$. Until recently, both firms had zero marginal costs. Recent environmental regulations have increased Firm 2's marginal cost to \$15. Firm 1's marginal cost remains constant at zero. True or false: As a result, the market price will rise to the *monopoly* level.

Surprisingly, this is true. However, it occurs only because the marginal cost for Firm 2 is \$15 or more. If the market were monopolized before the environmental regulations, the marginal revenue for the monopolist would be

$$MR = 30 - 2Q.$$

Profit maximization implies $MR = MC$, or $30 - 2Q = 0$. Therefore, $Q = 15$, and (using the demand curve) $P = \$15$.

The situation after the environmental regulations is a Cournot game where Firm 1's marginal costs are zero and Firm 2's marginal costs are \$15. We need to find the best response functions:

Firm 1's revenue is

$$PQ_1 = (30 - Q_1 - Q_2)Q_1 = 30Q_1 - Q_1^2 - Q_1Q_2,$$

and its marginal revenue is given by:

$$MR_1 = 30 - 2Q_1 - Q_2.$$

Profit maximization implies $MR_1 = MC_1$ or

$$30 - 2Q_1 - Q_2 = 0 \Rightarrow Q_1 = 15 - \frac{Q_2}{2},$$

which is Firm 1's best response function.

Firm 2's revenue function is symmetric to that of Firm 1 and hence

$$MR_2 = 30 - Q_1 - 2Q_2.$$

Profit maximization implies $MR_2 = MC_2$, or

$$30 - 2Q_2 - Q_1 = 15 \Rightarrow Q_2 = 7.5 - \frac{Q_1}{2},$$

which is Firm 2's best response function.

Cournot equilibrium occurs at the intersection of the best response functions. Substituting for Q_1 in the response function for Firm 2 yields:

$$Q_2 = 7.5 - 0.5 \left(15 - \frac{Q_2}{2} \right).$$

Thus $Q_2 = 0$ and $Q_1 = 15$. $P = 30 - Q_1 - Q_2 = \$15$, which is the monopoly price.

6. Suppose that two identical firms produce widgets and that they are the only firms in the market. Their costs are given by $C_1 = 60Q_1$ and $C_2 = 60Q_2$, where Q_1 is the output of Firm 1 and Q_2 the output of Firm 2. Price is determined by the following demand curve:

$$P = 300 - Q$$

where $Q = Q_1 + Q_2$.

- a. Find the Cournot-Nash equilibrium. Calculate the profit of each firm at this equilibrium.

Profit for Firm 1, $TR_1 - TC_1$, is equal to

$$\pi_1 = 300Q_1 - Q_1^2 - Q_1Q_2 - 60Q_1 = 240Q_1 - Q_1^2 - Q_1Q_2.$$

Therefore,

$$\frac{\partial \pi_1}{\partial Q_1} = 240 - 2Q_1 - Q_2.$$

Setting this equal to zero and solving for Q_1 in terms of Q_2 :

$$Q_1 = 120 - 0.5Q_2.$$

This is Firm 1's reaction function. Because Firm 2 has the same cost structure, Firm 2's reaction function is

$$Q_2 = 120 - 0.5Q_1.$$

Substituting for Q_2 in the reaction function for Firm 1, and solving for Q_1 , we find

$$Q_1 = 120 - (0.5)(120 - 0.5Q_1), \text{ or } Q_1 = 80.$$

By symmetry, $Q_2 = 80$. Substituting Q_1 and Q_2 into the demand equation to determine the equilibrium price:

$$P = 300 - 80 - 80 = \$140.$$

Substituting the values for price and quantity into the profit functions,

$$\pi_1 = (140)(80) - (60)(80) = \$6400, \text{ and}$$

$$\pi_2 = (140)(80) - (60)(80) = \$6400.$$

Therefore, profit is \$6400 for both firms in the Cournot-Nash equilibrium.

- b. Suppose the two firms form a cartel to maximize joint profits. How many widgets will be produced? Calculate each firm's profit.**

Given the demand curve is $P = 300 - Q$, the marginal revenue curve is $MR = 300 - 2Q$. Profit will be maximized by finding the level of output such that marginal revenue is equal to marginal cost:

$$300 - 2Q = 60, \text{ or } Q = 120.$$

When total output is 120, price will be \$180, based on the demand curve. Since both firms have the same marginal cost, they will split the total output equally, so they each produce 60 units.

Profit for each firm is:

$$\pi = 180(60) - 60(60) = \$7200.$$

- c. Suppose Firm 1 were the only firm in the industry. How would market output and Firm 1's profit differ from that found in part b above?**

If Firm 1 were the only firm, it would produce where marginal revenue is equal to marginal cost, as found in part b. In this case Firm 1 would produce the entire 120 units of output and earn a profit of \$14,400.

- d. Returning to the duopoly of part b, suppose Firm 1 abides by the agreement, but Firm 2 cheats by increasing production. How many widgets will Firm 2 produce? What will be each firm's profits?**

Assuming their agreement is to split the market equally, Firm 1 produces 60 widgets. Firm 2 cheats by producing its profit-maximizing level, given $Q_1 = 60$. Substituting $Q_1 = 60$ into Firm 2's reaction function:

$$Q_2 = 120 - \frac{60}{2} = 90.$$

Total industry output, Q_T , is equal to Q_1 plus Q_2 :

$$Q_T = 60 + 90 = 150.$$

Substituting Q_T into the demand equation to determine price:

$$P = 300 - 150 = \$150.$$

Substituting Q_1 , Q_2 , and P into the profit functions:

$$\pi_1 = (150)(60) - (60)(60) = \$5400, \text{ and}$$

$$\pi_2 = (150)(90) - (60)(90) = \$8100.$$

Firm 2 increases its profits at the expense of Firm 1 by cheating on the agreement.

- 7. Suppose that two competing firms, A and B, produce a homogeneous good. Both firms have a marginal cost of $MC = \$50$. Describe what would happen to output and price in each of the following situations if the firms are at (i) Cournot equilibrium, (ii) collusive equilibrium, and (iii) Bertrand equilibrium.**

- a. Because Firm A must increase wages, its MC increases to \$80.**

(i) In a Cournot equilibrium you must think about the effect on the reaction functions, as illustrated in Figure 12.5 of the text. When Firm A experiences an increase in marginal cost, its reaction function will shift inward. The quantity produced by Firm A will decrease and the

quantity produced by Firm *B* will increase. Total quantity produced will decrease and price will increase.

- (ii) In a collusive equilibrium, the two firms will collectively act like a monopolist. When the marginal cost of Firm *A* increases, Firm *A* will reduce its production to zero, because Firm *B* can produce at a lower marginal cost. Because Firm *B* can produce the entire industry output at a marginal cost of \$50, there will be no change in output or price. However, the firms will have to come to some agreement on how to share the profit earned by *B*.
- (iii) Before the increase in Firm *A*'s costs, both firms would charge a price equal to marginal cost ($P = \$50$) because the good is homogeneous. After Firm *A*'s marginal cost increases, Firm *B* will raise its price to \$79.99 (or some price just below \$80) and take all sales away from Firm *A*. Firm *A* would lose money on each unit sold at any price below its marginal cost of \$80, so it will produce nothing.

b. The marginal cost of both firms increases.

- (i) Again refer to Figure 12.5. The increase in the marginal cost of both firms will shift both reaction functions inward. Both firms will decrease quantity produced and price will increase.
- (ii) When marginal cost increases, both firms will produce less and price will increase, as in the monopoly case.
- (iii) Price will increase to the new level of marginal cost and quantity will decrease.

c. The demand curve shifts to the right.

- (i) This is the opposite of the case in part b. In this situation, both reaction functions will shift outward and both will produce a higher quantity. Price will tend to increase.
- (ii) Both firms will increase the quantity produced as demand and marginal revenue increase. Price will also tend to increase.
- (iii) Both firms will supply more output. Given that marginal cost remains the same, the price will not change.

8. Suppose the airline industry consisted of only two firms: American and Texas Air Corp. Let the two firms have identical cost functions, $C(q) = 40q$. Assume the demand curve for the industry is given by $P = 100 - Q$ and that each firm expects the other to behave as a Cournot competitor.

a. Calculate the Cournot-Nash equilibrium for each firm, assuming that each chooses the output level that maximizes its profits when taking its rival's output as given. What are the profits of each firm?

First, find the reaction function for each firm; then solve for price, quantity, and profit. Profit for Texas Air, π_1 , is equal to total revenue minus total cost:

$$\begin{aligned}\pi_1 &= (100 - Q_1 - Q_2)Q_1 - 40Q_1, \text{ or} \\ \pi_1 &= 100Q_1 - Q_1^2 - Q_1Q_2 - 40Q_1, \text{ or } \pi_1 = 60Q_1 - Q_1^2 - Q_1Q_2.\end{aligned}$$

The change in π_1 with respect to Q_1 is

$$\frac{\partial \pi_1}{\partial Q_1} = 60 - 2Q_1 - Q_2.$$

Setting the derivative to zero and solving for Q_1 gives Texas Air's reaction function:

$$Q_1 = 30 - 0.5Q_2.$$

Because American has the same cost structure, American's reaction function is

$$Q_2 = 30 - 0.5Q_1.$$

Substituting for Q_2 in the reaction function for Texas Air,

$$Q_1 = 30 - 0.5(30 - 0.5Q_1), \text{ or } Q_1 = 20.$$

By symmetry, $Q_2 = 20$. Industry output, Q_T , is Q_1 plus Q_2 , or

$$Q_T = 20 + 20 = 40.$$

Substituting industry output into the demand equation, we find $P = \$60$. Substituting Q_1 , Q_2 , and P into the profit function, we find

$$\pi_1 = \pi_2 = 60(20) - 20^2 - (20)(20) = \$400.$$

- b. What would be the equilibrium quantity if Texas Air had constant marginal and average costs of \$25 and American had constant marginal and average costs of \$40?**

By solving for the reaction functions under this new cost structure, we find that profit for Texas Air is equal to

$$\pi_1 = 100Q_1 - Q_1^2 - Q_1Q_2 - 25Q_1 = 75Q_1 - Q_1^2 - Q_1Q_2.$$

The change in profit with respect to Q_1 is

$$\frac{\partial \pi_1}{\partial Q_1} = 75 - 2Q_1 - Q_2.$$

Set the derivative to zero, and solve for Q_1 in terms of Q_2 ,

$$Q_1 = 37.5 - 0.5Q_2.$$

This is Texas Air's reaction function. Since American has the same cost structure as in part a, American's reaction function is the same as before:

$$Q_2 = 30 - 0.5Q_1.$$

To determine Q_1 , substitute for Q_2 in the reaction function for Texas Air and solve for Q_1 :

$$Q_1 = 37.5 - (0.5)(30 - 0.5Q_1), \text{ so } Q_1 = 30.$$

Texas Air finds it profitable to increase output in response to a decline in its cost structure.

To determine Q_2 , substitute for Q_1 in the reaction function for American:

$$Q_2 = 30 - (0.5)(30) = 15.$$

American has cut back slightly in its output in response to the increase in output by Texas Air.

Total quantity, Q_T , is $Q_1 + Q_2$, or

$$Q_T = 30 + 15 = 45.$$

Compared to part a, the equilibrium quantity has risen slightly.

- c. Assuming that both firms have the original cost function, $C(q) = 40q$, how much should Texas Air be willing to invest to lower its marginal cost from 40 to 25, assuming that American will not follow suit? How much should American be willing to spend to reduce its marginal cost to 25, assuming that Texas Air will have marginal costs of 25 regardless of American's actions?**

Recall that profits for both firms were \$400 under the original cost structure. With constant average and marginal costs of \$25, we determined in part b that Texas Air would produce 30 units and American 15. Industry price would then be $P = 100 - 30 - 15 = \$55$. Texas Air's profits would be

$$(55)(30) - (25)(30) = \$900.$$

The difference in profit is \$500. Therefore, Texas Air should be willing to invest up to \$500 to lower costs from 40 to 25 per unit (assuming American does not follow suit).

To determine how much American would be willing to spend to reduce its average costs, calculate the difference in American's profits, assuming Texas Air's average cost is \$25. First, without investment, American's profits would be:

$$(55)(15) - (40)(15) = \$225.$$

Second, with investment by both firms, the reaction functions would be:

$$Q_1 = 37.5 - 0.5Q_2 \text{ and}$$

$$Q_2 = 37.5 - 0.5Q_1.$$

To determine Q_1 , substitute for Q_2 in the first reaction function and solve for Q_1 :

$$Q_1 = 37.5 - (0.5)(37.5 - 0.5Q_1), \text{ which implies } Q_1 = 25.$$

Since the firms are symmetric, Q_2 is also 25.

Substituting industry output into the demand equation to determine price:

$$P = 100 - 50 = \$50.$$

Therefore, American's profits when both firms have $MC = AC = 25$ are

$$\pi_2 = (50)(25) - (25)(25) = \$625.$$

The difference in profit with and without the cost-saving investment for American is \$400. American would be willing to invest up to \$400 to reduce its marginal cost to 25 if Texas Air also has marginal costs of 25.

9. Demand for light bulbs can be characterized by $Q = 100 - P$, where Q is in millions of boxes of lights sold and P is the price per box. There are two producers of lights, Everglow and Dimlit. They have identical cost functions:

$$C_i = 10Q_i + \frac{1}{2}Q_i^2 \quad (i = E, D) \qquad Q = Q_E + Q_D.$$

- a. Unable to recognize the potential for collusion, the two firms act as short-run perfect competitors. What are the equilibrium values of Q_E , Q_D , and P ? What are each firm's profits?

Given that the total cost function is $C_i = 10Q_i + 1/2Q_i^2$, the marginal cost curve for each firm is $MC_i = 10 + Q_i$. In the short run, perfectly competitive firms determine the optimal level of output by taking price as given and setting price equal to marginal cost. There are two ways to solve this problem. One way is to set price equal to marginal cost for each firm so that:

$$P = 100 - Q_1 - Q_2 = 10 + Q_1$$

$$P = 100 - Q_1 - Q_2 = 10 + Q_2.$$

Given we now have two equations and two unknowns, we can solve for Q_1 and Q_2 simultaneously. Solve the second equation for Q_2 to get

$$Q_2 = \frac{90 - Q_1}{2},$$

and substitute into the other equation to get

$$100 - Q_1 - \frac{90 - Q_1}{2} = 10 + Q_1.$$

This yields a solution where $Q_1 = 30$, $Q_2 = 30$, and $P = \$40$. You can verify that $P = MC$ for each firm. Profit is total revenue minus total cost or

$$\pi_i = 40(30) - [10(30) + 0.5(30)^2] = \$450 \text{ million.}$$

The other way to solve the problem is to find the market supply curve by summing the marginal cost curves, which yields $Q_M = 2P - 20$. Set supply equal to demand to find $P = \$40$ and $Q = 60$ in the market, or 30 per firm since they are identical.

- b. Top management in both firms is replaced. Each new manager independently recognizes the oligopolistic nature of the light bulb industry and plays Cournot. What are the equilibrium values of Q_E , Q_D , and P ? What are each firm's profits?**

To determine the Cournot-Nash equilibrium, we first calculate the reaction function for each firm, then solve for price, quantity, and profit. Profits for Everglow are equal to $TR_E - TC_E$, or

$$\pi_E = (100 - Q_E - Q_D)Q_E - (10Q_E + 0.5Q_E^2) = 90Q_E - 1.5Q_E^2 - Q_EQ_D.$$

The change in profit with respect to Q_E is

$$\frac{\partial \pi_E}{\partial Q_E} = 90 - 3Q_E - Q_D.$$

To determine Everglow's reaction function, set the change in profits with respect to Q_E equal to 0 and solve for Q_E :

$$90 - 3Q_E - Q_D = 0, \text{ or}$$

$$Q_E = \frac{90 - Q_D}{3}.$$

Because Dimlit has the same cost structure, Dimlit's reaction function is

$$Q_D = \frac{90 - Q_E}{3}.$$

Substituting for Q_D in the reaction function for Everglow, and solving for Q_E :

$$Q_E = \frac{90 - \frac{90 - Q_E}{3}}{3}$$

$$3Q_E = 90 - 30 + \frac{Q_E}{3}$$

$$Q_E = 22.5.$$

By symmetry, $Q_D = 22.5$, and total industry output is 45.

Substituting industry output into the demand equation gives P :

$$45 = 100 - P, \text{ or } P = \$55.$$

Each firm's profit equals total revenue minus total cost:

$$B_i = 55(22.5) - [10(22.5) + 0.5(22.5)^2] = \$759.4 \text{ million.}$$

- c. **Suppose the Everglow manager guesses correctly that Dimlit is playing Cournot, so Everglow plays Stackelberg. What are the equilibrium values of Q_E , Q_D , and P ? What are each firm's profits?**

Recall Everglow's profit function:

$$\pi_E = (100 - Q_E - Q_D)Q_E - (10Q_E + 0.5Q_E^2).$$

If Everglow sets its quantity first, knowing Dimlit's reaction function (i.e., $Q_D = 30 - \frac{Q_E}{3}$), we may determine Everglow's profit by substituting for Q_D in its profit function. We find

$$\pi_E = 60Q_E - \frac{7Q_E^2}{6}.$$

To determine the profit-maximizing quantity, differentiate profit with respect to Q_E , set the derivative to zero and solve for Q_E :

$$\frac{\partial \pi_E}{\partial Q_E} = 60 - \frac{7Q_E}{3} = 0, \text{ or } Q_E = 25.7.$$

Substituting this into Dimlit's reaction function, $Q_D = 30 - \frac{25.7}{3} = 21.4$. Total industry output is therefore 47.1 and $P = \$52.90$. Profit for Everglow is

$$\pi_E = (52.90)(25.7) - [10(25.7) + 0.5(25.7)^2] = \$772.3 \text{ million.}$$

Profit for Dimlit is

$$\pi_D = (52.90)(21.4) - [10(21.4) + 0.5(21.4)^2] = \$689.1 \text{ million.}$$

- d. **If the managers of the two companies collude, what are the equilibrium values of Q_E , Q_D , and P ? What are each firm's profits?**

Because the firms are identical, they should split the market equally, so each produces $Q/2$ units, where Q is the total industry output. Each firm's total cost is therefore

$$C_i = 10\left(\frac{Q}{2}\right) + \frac{1}{2}\left(\frac{Q}{2}\right)^2,$$

and total industry cost is

$$TC = 2C_i = 10Q + \left(\frac{Q}{2}\right)^2.$$

Hence, industry marginal cost is

$$MC = 10 + 0.5Q.$$

With inverse industry demand given by $P = 100 - Q$, industry marginal revenue is

$$MR = 100 - 2Q.$$

Setting $MR = MC$, we have

$$100 - 2Q = 10 + 0.5Q, \text{ and so } Q = 36,$$

which means $Q_E = Q_D = Q/2 = 18$.

Substituting Q in the demand equation to determine price:

$$P = 100 - 36 = \$64.$$

The profit for each firm is equal to total revenue minus total cost:

$$\pi_i = 64(18) - [10(18) + 0.5(18)^2] = \$810 \text{ million.}$$

Note that you can also solve for the optimal quantities by treating the two firms as a monopolist with two plants. In that case, the optimal outputs satisfy the condition $MR = MC_E = MC_D$. Setting marginal revenue equal to each marginal cost function gives the following two equations:

$$MR = 100 - 2(Q_E + Q_D) = 10 + Q_E = MC_E$$

$$MR = 100 - 2(Q_E + Q_D) = 10 + Q_D = MC_D.$$

Solving simultaneously, we get the same solution as before; that is, $Q_E = Q_D = 18$.

- 10. Two firms produce luxury sheepskin auto seat covers, Western Where (WW) and B.B.B. Sheep (BBBS). Each firm has a cost function given by**

$$C(q) = 30q + 1.5q^2$$

The market demand for these seat covers is represented by the inverse demand equation

$$P = 300 - 3Q$$

where $Q = q_1 + q_2$, total output.

- a. If each firm acts to maximize its profits, taking its rival's output as given (i.e., the firms behave as Cournot oligopolists), what will be the equilibrium quantities selected by each firm? What is total output, and what is the market price? What are the profits for each firm?**

Find the best response functions (the reaction curves) for both firms by setting marginal revenue equal to marginal cost (alternatively you can set up the profit function for each firm and differentiate with respect to the quantity produced for that firm):

$$R_1 = P q_1 = (300 - 3(q_1 + q_2)) q_1 = 300q_1 - 3q_1^2 - 3q_1q_2.$$

$$MR_1 = 300 - 6q_1 - 3q_2$$

$$MC_1 = 30 + 3q_1$$

$$300 - 6q_1 - 3q_2 = 30 + 3q_1$$

$$q_1 = 30 - (1/3)q_2.$$

By symmetry, BBBS's best response function will be:

$$q_2 = 30 - (1/3)q_1.$$

Cournot equilibrium occurs at the intersection of these two best response functions, which is:

$$q_1 = q_2 = 22.5.$$

Thus,

$$Q = q_1 + q_2 = 45$$

$$P = 300 - 3(45) = \$165.$$

Profit for both firms will be equal and given by:

$$\pi = R - C = (165)(22.5) - [30(22.5) + 1.5(22.5^2)] = \$2278.13.$$

- b. It occurs to the managers of WW and BBBS that they could do a lot better by colluding. If the two firms collude, what will be the profit-maximizing choice of output? The industry price? The output and the profit for each firm in this case?**

In this case the firms should each produce half the quantity that maximizes total industry profits (i.e., half the monopoly output). Note that if the two firms had different cost functions, then it would not be optimal for them to split the monopoly output evenly.

Joint profits will be $(300 - 3Q)Q - 2[30(Q/2) + 1.5(Q/2)^2] = 270Q - 3.75Q^2$, which will be maximized at $Q = 36$. You can find this quantity by differentiating the profit function with respect to Q , setting the derivative equal to zero, and solving for Q : $d\pi/dQ = 270 - 7.5Q = 0$, so $Q = 36$.

The optimal output for each firm is $q_1 = q_2 = 36/2 = 18$, and the optimal price for the firms to charge is $P = 300 - 3(36) = \$192$.

Profit for each firm will be $\pi = (192)(18) - [30(18) + 1.5(18^2)] = \2430 .

- c. The managers of these firms realize that explicit agreements to collude are illegal. Each firm must decide on its own whether to produce the Cournot quantity or the cartel quantity. To aid in making the decision, the manager of WW constructs a payoff matrix like the one below. Fill in each box with the profit of WW and the profit of BBBS. Given this payoff matrix, what output strategy is each firm likely to pursue?**

To fill in the payoff matrix, we have to calculate the profit each firm would make with each of the possible output level combinations. We already know the profits if both choose the Cournot output or both choose the cartel output. If WW produces the Cournot level of output (22.5) and BBBS produces the collusive level (18), then:

$$Q = q_1 + q_2 = 22.5 + 18 = 40.5$$

$$P = 300 - 3(40.5) = \$178.50.$$

$$\text{Profit for WW} = (178.5)(22.5) - [30(22.5) + 1.5(22.5^2)] = \$2581.88.$$

$$\text{Profit for BBBS} = (178.5)(18) - [30(18) + 1.5(18^2)] = \$2187.$$

If WW chooses the collusive output level and BBBS chooses the Cournot output, profits will be reversed. Rounding off profits to whole dollars, the payoff matrix is as follows.

Profit Payoff Matrix (WW profit, BBBS profit)		BBBS	
		Produce Cournot q	Produce Cartel q
W	Produce Cournot q	2278, 2278	2582, 2187
W	Produce Cartel q	2187, 2582	2430, 2430

For each firm, the Cournot output dominates the cartel output, because each firm's profit is higher when it chooses the Cournot output, *regardless of the other firm's output*. For example, if WW chooses the Cournot output, BBBS earns \$2278 if it chooses the Cournot output but only \$2187 if it chooses the cartel output. On the other hand, if WW chooses the cartel output, BBBS earns \$2582 with the Cournot output, which is better than the \$2430 profit it would make with the cartel output. So no matter what WW chooses, BBBS is always better off choosing the Cournot output. Therefore, producing at the Cournot output levels will be the Nash Equilibrium in this industry.

This is a prisoners' dilemma game, because both firms would make greater profits if they both produced the cartel output. The cartel profit of \$2430 is greater than the Cournot profit of \$2278. The problem is that each firm has an incentive to cheat and produce the Cournot output instead of the cartel output. For example, if the firms are colluding and WW continues to produce the cartel output but BBBS increases output to the Cournot level, BBBS increases its profit from \$2430 to \$2582. When both firms do this, however, they wind up back at the Nash-Cournot equilibrium where each produces the Cournot output level and each makes a profit of only \$2278.

- d. **Suppose WW can set its output level *before* BBBS does. How much will WW choose to produce in this case? How much will BBBS produce? What is the market price, and what is the profit for each firm? Is WW better off by choosing its output first? Explain why or why not.**

WW will use the Stackelberg strategy. WW knows that BBBS will choose a quantity q_2 , which will be its best response to q_1 or:

$$q_2 = 30 - \frac{1}{3}q_1.$$

WW's profits will be:

$$\pi = Pq_1 - C_1 = (300 - 3q_1 - 3q_2)q_1 - (30q_1 + 1.5q_1^2)$$

$$\pi = Pq_1 - C_1 = (300 - 3q_1 - 3(30 - \frac{1}{3}q_1))q_1 - (30q_1 + 1.5q_1^2)$$

$$\pi = 180q_1 - 3.5q_1^2$$

Profit maximization implies:

$$\frac{d\pi}{dq_1} = 180 - 7q_1 = 0.$$

This results in $q_1 = 25.7$ and $q_2 = 21.4$. The equilibrium price and profits will be:

$$P = 300 - 3(q_1 + q_2) = 300 - 3(25.7 + 21.4) = \$158.70$$

$$\pi_1 = (158.70)(25.7) - [(30)(25.7) + 1.5(25.7)^2] = \$2316.86$$

$$\pi_2 = (158.70)(21.4) - [(30)(21.4) + 1.5(21.4)^2] = \$2067.24.$$

WW is able to benefit from its first-mover advantage by committing to a high level of output. Since BBBS moves after WW has selected its output, BBBS can only react to the output decision of WW. If WW produces its Cournot output as a leader, BBBS produces its Cournot output as a follower. Hence WW cannot do worse as a leader than it does in the Cournot game. When WW produces more, BBBS produces less, raising WW's profits.

11. Two firms compete by choosing price. Their demand functions are

$$Q_1 = 20 - P_1 + P_2 \quad \text{and} \quad Q_2 = 20 + P_1 - P_2$$

where P_1 and P_2 are the prices charged by each firm, respectively, and Q_1 and Q_2 are the resulting demands. Note that the demand for each good depends only on the difference in prices; if the two firms colluded and set the same price, they could make that price as high as they wanted, and earn infinite profits. Marginal costs are zero.

- a. Suppose the two firms set their prices at the *same time*. Find the resulting Nash equilibrium. What price will each firm charge, how much will it sell, and what will its profit be? (*Hint: Maximize the profit of each firm with respect to its price.*)

To determine the Nash equilibrium in prices, first calculate the reaction function for each firm, then solve for price. With zero marginal cost, profit for Firm 1 is:

$$\pi_1 = P_1 Q_1 = P_1(20 - P_1 + P_2) = 20P_1 - P_1^2 + P_2 P_1.$$

Marginal revenue is the slope of the total revenue function (here it is the derivative of the profit function with respect to P_1 because total cost is zero):

$$MR_1 = 20 - 2P_1 + P_2.$$

At the profit-maximizing price, $MR_1 = 0$. Therefore,

$$P_1 = \frac{20 + P_2}{2}.$$

This is Firm 1's reaction function. Because Firm 2 is symmetric to Firm 1, its reaction function is $P_2 = \frac{20 + P_1}{2}$. Substituting Firm 2's reaction function into that of Firm 1:

$$P_1 = \frac{20 + \frac{20 + P_1}{2}}{2} = 10 + 5 + \frac{P_1}{4}, \text{ so } P_1 = \$20$$

By symmetry, $P_2 = \$20$.

To determine the quantity produced by each firm, substitute P_1 and P_2 into the demand functions:

$$Q_1 = 20 - 20 + 20 = 20, \text{ and}$$

$$Q_2 = 20 + 20 - 20 = 20.$$

Profits for Firm 1 are $P_1 Q_1 = \$400$, and, by symmetry, profits for Firm 2 are also \$400.

- b. Suppose Firm 1 sets its price *first* and then Firm 2 sets its price. What price will each firm charge, how much will it sell, and what will its profit be?

If Firm 1 sets its price first, it takes Firm 2's reaction function into account. Firm 1's profit function is:

$$\pi_1 = P_1 \left(20 - P_1 + \frac{20 + P_1}{2} \right) = 30P_1 - \frac{P_1^2}{2}.$$

To determine the profit-maximizing price, find the change in profit with respect to a change in price:

$$\frac{d\pi_1}{dP_1} = 30 - P_1.$$

Set this expression equal to zero to find the profit-maximizing price:

$$30 - P_1 = 0, \text{ or } P_1 = \$30.$$

Substitute P_1 in Firm 2's reaction function to find P_2 :

$$P_2 = \frac{20 + 30}{2} = \$25.$$

At these prices,

$$Q_1 = 20 - 30 + 25 = 15, \text{ and}$$

$$Q_2 = 20 + 30 - 25 = 25.$$

Profits are

$$\pi_1 = (30)(15) = \$450 \text{ and}$$

$$\pi_2 = (25)(25) = \$625.$$

If Firm 1 must set its price first, Firm 2 is able to undercut Firm 1 and gain a larger market share. However, both firms make greater profits than they did in part a, where they chose prices simultaneously.

- c. **Suppose you are one of these firms and that there are three ways you could play the game: (i) Both firms set price at the same time; (ii) You set price first; or (iii) Your competitor sets price first. If you could choose among these options, which would you prefer? Explain why.**

Compare the Nash profits in part a, \$400, with the profits in part b, \$450 for the firm that sets price first and \$625 for the follower. Clearly it is best to be the follower, so you should choose option (iii). From the reaction functions, we know that the price leader raises price and provokes a price increase by the follower. By being able to move second, however, the follower increases price by less than the leader, and hence undercuts the leader. Both firms enjoy increased profits, but the follower does better.

12. **The dominant firm model can help us understand the behavior of some cartels. Let's apply this model to the OPEC oil cartel. We will use isoelastic curves to describe world demand W and noncartel (competitive) supply S . Reasonable numbers for the price elasticities of world demand and noncartel supply are $-1/2$ and $1/2$, respectively. Then, expressing W and S in millions of barrels per day (mb/d), we could write**

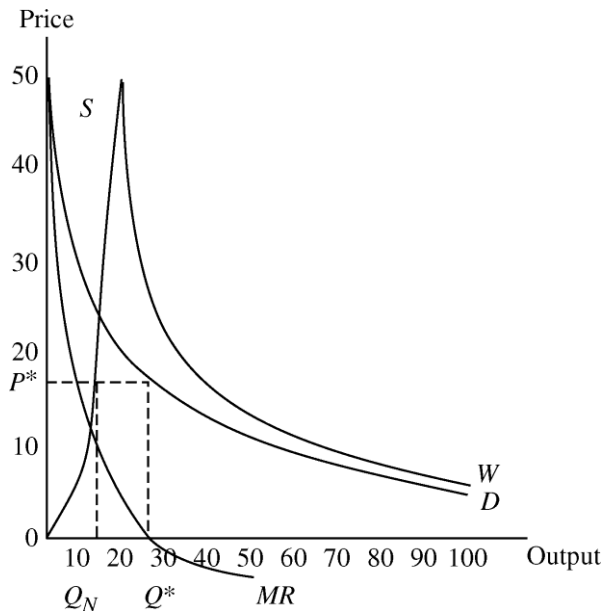
$$W = 160P^{-\frac{1}{2}} \quad \text{and} \quad S = 3\frac{1}{3}P^{\frac{1}{2}}.$$

Note that OPEC's net demand is $D = W - S$.

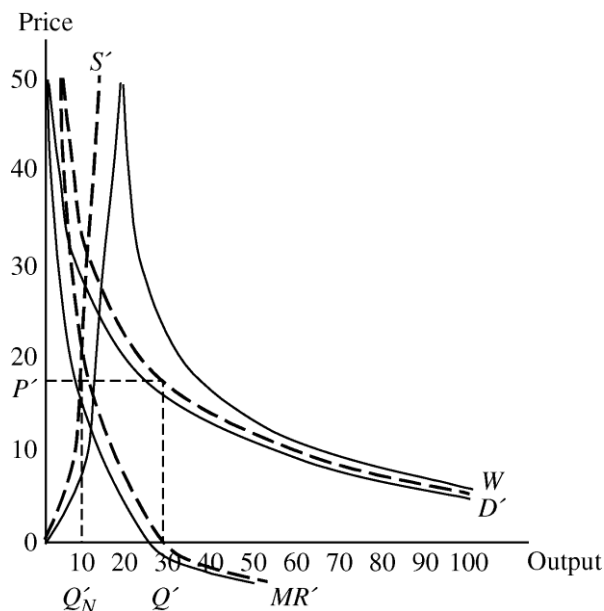
- a. Draw the world demand curve W , the non-OPEC supply curve S , OPEC's net demand curve D , and OPEC's marginal revenue curve. For purposes of approximation, assume OPEC's production cost is zero. Indicate OPEC's optimal price, OPEC's optimal production, and non-OPEC production on the diagram. Now, show on the diagram how the various curves will shift and how OPEC's optimal price will change if non-OPEC supply becomes more expensive because reserves of oil start running out.

OPEC's initial net demand curve is $D = 160P^{-1/2} - 3\frac{1}{3}P^{1/2}$. Marginal revenue is quite difficult to find. If you were going to determine it analytically, you would have to solve OPEC's net demand curve for P . Then take that expression and multiply by $Q (=D)$ to get total revenue as a function of output. Finally, you would take the derivative of revenue with respect to Q . The MR curve would look approximately like that shown in the figure below.

OPEC's optimal production, Q^* , occurs where $MR = 0$ (since production cost is assumed to be zero), and OPEC's optimal price, P^* , is found from the net demand curve at Q^* . Non-OPEC production, Q_N , can be read off the non-OPEC supply curve, S , at price P^* .



Now, if non-OPEC oil becomes more expensive, the supply curve S shifts to S' . This shifts OPEC's net demand curve outward from D to D' , which in turn creates a new marginal revenue curve, MR' , and a new optimal OPEC production level of Q' , yielding a new higher price of P' . At this new price, non-OPEC production is Q_N' . The new S , D , and MR curves are dashed lines. Unfortunately, the diagram is difficult to sort out, but OPEC's new optimal output has increased to around 30, non-OPEC supply has dropped to about 10, and the optimal price has increased slightly.



- b. Calculate OPEC's optimal (profit-maximizing) price. (*Hint: Because OPEC's cost is zero, just write the expression for OPEC revenue and find the price that maximizes it.*)

Since costs are zero, OPEC will choose a price that maximizes total revenue:

$$\text{Max } \pi = PQ = P(W - S)$$

$$\pi = P \left(160P^{-1/2} - 3\frac{1}{3}P^{1/2} \right) = 160P^{1/2} - 3\frac{1}{3}P^{3/2}.$$

To determine the profit-maximizing price, take the derivative of profit with respect to price and set it equal to zero:

$$\frac{\partial \pi}{\partial P} = 80P^{-1/2} - \left(3\frac{1}{3} \right) \left(\frac{3}{2} \right) P^{1/2} = 80P^{-1/2} - 5P^{1/2} = 0.$$

Solving for P ,

$$5P^{1/2} = \frac{80}{P^{1/2}}, \text{ or } P = \$16.$$

At this price, $W = 40$, $S = 13.33$, and $D = 26.67$ as shown in the first diagram.

- c. Suppose the oil-consuming countries were to unite and form a "buyers' cartel" to gain monopsony power. What can we say, and what can't we say, about the impact this action would have on price?

If the oil-consuming countries unite to form a buyers' cartel, then we have a monopoly (OPEC) facing a monopsony (the buyers' cartel). As a result, there are no well-defined demand or supply curves. We expect that the price will fall below the monopoly price when the buyers also collude, because monopsony power offsets some monopoly power. However, economic theory cannot determine the exact price that results from this bilateral monopoly because the price depends on the bargaining skills of the two parties, as well as on other factors such as the elasticities of supply and demand.

13. Suppose the market for tennis shoes has one dominant firm and five fringe firms. The market demand is $Q = 400 - 2P$. The dominant firm has a constant marginal cost of 20. The fringe firms each have a marginal cost of $MC = 20 + 5q$.

a. Verify that the total supply curve for the five fringe firms is $Q_f = P - 20$.

The total supply curve for the five firms is found by horizontally summing the five marginal cost curves, or in other words, adding up the quantity supplied by each firm for any given price. Rewrite each fringe firm's marginal cost curve as follows:

$$MC = 20 + 5q = P$$

$$5q = P - 20$$

$$q = \frac{P}{5} - 4$$

Since each firm is identical, the supply curve is five times the supply of one firm for any given price:

$$Q_f = 5\left(\frac{P}{5} - 4\right) = P - 20.$$

b. Find the dominant firm's demand curve.

The dominant firm's demand curve is given by the difference between the market demand and the fringe total supply curve:

$$Q_D = 400 - 2P - (P - 20) = 420 - 3P.$$

c. Find the profit-maximizing quantity produced and price charged by the dominant firm, and the quantity produced and price charged by each of the fringe firms.

The dominant firm will set marginal revenue equal to marginal cost. The marginal revenue curve has the same intercept and twice the slope of the linear inverse demand curve, which is shown below:

$$Q_D = 420 - 3P$$

$$P = 140 - \frac{1}{3}Q_D$$

$$MR = 140 - \frac{2}{3}Q_D.$$

Now set marginal revenue equal to marginal cost to find the profit-maximizing quantity for the dominant firm, and the price charged by the dominant firm:

$$MR = 140 - \frac{2}{3}Q_D = 20 = MC$$

$$Q_D = 180, \text{ and } P = \$80.$$

Each fringe firm will charge the same \$80 price as the dominant firm, and the total output produced by the five fringe firms will be $Q_f = P - 20 = 60$. Each fringe firm will therefore produce 12 units.

d. Suppose there are ten fringe firms instead of five. How does this change your results?

We need to find the new fringe supply curve, dominant firm demand curve, and dominant firm marginal revenue curve as above. The new total fringe supply curve is $Q_f = 2P - 40$. The new dominant firm demand curve is $Q_D = 440 - 4P$. The new dominant firm marginal revenue curve is $MR = 110 - \frac{Q}{2}$. The dominant firm will produce where marginal revenue is equal to marginal cost which occurs at 180 units. Substituting a quantity of 180 into the demand curve faced by the

dominant firm results in a price of \$65. Substituting the price of \$65 into the total fringe supply curve results in a total fringe quantity supplied of 90, so that each fringe firm will produce 9 units. Increasing the number of fringe firms reduces market price from \$80 to \$65, increases total market output from 240 to 270 units, and reduces the market share of the dominant firm from 75% to 67% (although the dominant firm continues to sell 180 units).

- e. **Suppose there continue to be five fringe firms but that each manages to reduce its marginal cost to $MC = 20 + 2q$. How does this change your results?**

Follow the same method as in earlier parts of this problem. Rewrite the fringe marginal cost curve as

$$q = \frac{P}{2} - 10.$$

The new total fringe supply curve is five times the individual fringe supply curve, which is also the fringe marginal cost curve:

$$Q_f = \frac{5}{2}P - 50.$$

The new dominant firm demand curve is found by subtracting the fringe supply curve from the market demand curve to get $Q_D = 450 - 4.5P$.

The new inverse demand curve for the dominant firm is therefore,

$$P = 100 - \frac{Q}{4.5}.$$

The dominant firm's new marginal revenue curve is

$$MR = 100 - \frac{2Q}{4.5}.$$

Set $MR = MC = 20$. The dominant firm will produce 180 units and will charge a price of

$$P = 100 - \frac{180}{4.5} = \$60.$$

Therefore, price drops from \$80 to \$60. The fringe firms will produce a total of $\frac{5}{2}(60) - 50 = 100$ units, so total industry output increases from 240 to 280. The market share of the dominant firm drops from 75% to 64%.

14. **A lemon-growing cartel consists of four orchards. Their total cost functions are:**

$$TC_1 = 20 + 5Q_1^2$$

$$TC_2 = 25 + 3Q_2^2$$

$$TC_3 = 15 + 4Q_3^2$$

$$TC_4 = 20 + 6Q_4^2$$

TC is in hundreds of dollars, and Q is in cartons per month picked and shipped.

- a. **Tabulate total, average, and marginal costs for each firm for output levels between 1 and 5 cartons per month (i.e., for 1, 2, 3, 4, and 5 cartons).**

The following tables give total, average, and marginal costs for each firm.

Units	Firm 1			Firm 2		
	<i>TC</i>	<i>AC</i>	<i>MC</i>	<i>TC</i>	<i>AC</i>	<i>MC</i>
0	20	—	—	25	—	—
1	25	25	5	28	28	3
2	40	20	15	37	18.5	9
3	65	21.67	25	52	17.33	15
4	100	25	35	73	18.25	21
5	145	29	45	100	20	27

Units	Firm 3			Firm 4		
	<i>TC</i>	<i>AC</i>	<i>MC</i>	<i>TC</i>	<i>AC</i>	<i>MC</i>
0	15	—	—	20	—	—
1	19	19	4	26	26	6
2	31	15.5	12	44	22	18
3	51	17	20	74	24.67	30
4	79	19.75	28	116	29	42
5	115	23	36	170	34	54

- b. If the cartel decided to ship 10 cartons per month and set a price of \$25 per carton, how should output be allocated among the firms?

The cartel should assign production such that the lowest marginal cost is achieved for each unit, i.e.,

Cartel Unit Assigned	Firm Assigned	<i>MC</i>
1	2	3
2	3	4
3	1	5
4	4	6
5	2	9
6	3	12
7	1	15
8	2	15
9	4	18
10	3	20

Therefore, Firms 1 and 4 produce two units each and Firms 2 and 3 produce three units each.

- c. At this shipping level, which firm has the most incentive to cheat? Does any firm *not* have an incentive to cheat?**

At this level of output, Firm 2 has the lowest marginal cost for producing one more unit beyond its allocation, i.e., $MC = 21$ for the fourth unit for Firm 2. In addition, $MC = 21$ is less than the price of \$25. For all other firms, the next unit has a marginal cost equal to or greater than \$25. Firm 2 has the most incentive to cheat, while Firms 3 and 4 have no incentive to cheat, and Firm 1 is indifferent.